DYNAMIC MODELING AND ANALYSIS OF A THREE – DIMENTIONAL OVERHEAD CRANE SYSTEM WITH THE VARIATION OF LOAD MASS AND HOISTING/LOWERING FORCE

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ABSTRACT

Cranes are commonly used in the industry, in the military to move heavy loads, or assembly of large structures. Three basic movements of the crane is moving vertically, horizontally and lifting loads. However, the vibration of the load during move affects the safety and operational efficiency of the system. The velocity escalation to enhance performance as the vibration is caused by losing of time and counterproductive. This paper proposes solutions to improve the efficiency of the crane in conditions of appropriate parameters. A dynamic model of the overhead crane system is also developed in three-dimensional space based on Euler- Lagrange method, including the description of the movement of the load in the vertical, horizontal and lifting direction. Effects of parameters variation as load mass, hoisting/ lowering force on the response of the system on the time domain and frequency domain are discussed through simulation results. The article also suggestes the parameter range to work effectively. Finally, some conclusions are presented

Keywords: *Dynamical models; 3D crane, Euler- Lagrange method; time domain and frequency domain, power spectral density, effective parameter range*

INTRODUCTION

Overhead crane systems in three-dimensional (3-D crane) often used to transport heavy loads in factories and habors.... During speed acceleration or reduction always cause unwanted load swing at the destination location. Disturbances such as friction, wind and rain also reduces performance overhead cranes, it adversely impacts on the crane performance. These problems reduce the efficiency of work. In some cases, they cause damages to the load or become unsafe. Therefore, the divelopement and analysis of dynamic models with the change of crane parameters is necessary to promote the working efficiency of the crane.

The mathematical description and nonlinear control as the crane was studied from the early age [8,10,11,13,14]. The development of a nonlinear dynamical models and methods for crane control 2-D, 3-D have been written in many reports [1,6-8]. Most of the reports focuse on the issue of handling to minimize vibration loads [2,4,5,9]. In those studies, the kinematic equations of complex nonlinear systems for cranes have been analyzed to optimize the direction controls. From the anti-vibration control by rational design of mechanical components or signal [3,12], analysis of the impact of these parameters [4,5,6], to designing controllers based on theory of the modern control [5,6]. In published reports, the authors focused on solutions to design controllers or analyzed the influence of system parameters on the time domain. This study presents a general model of the crane and the kinetic equation of the crane system in three-dimensional space. Euler-Lagrange principle is applied to describe the kinetics of the system The simulation algorithm is implemented in Matlab. Responses of trolley positions, swing angles of the system and the power spectral density are obtained in both time domain and frequency domain. The effect of payloads and hoisting force by varying these two parameters are presented. Simulation results are analyzed and concluded.

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MODELING OF A THREE DIMENTIONAL OVERHEAD CRANE

Figure 1 describes the coordinate system of a 3-D crane and its load. *XYZ* is set as a fixed coordinate system and $X_cY_cZ_c$ as trolleys. The axis of the trolley coordinate system are paralleled respectively fixed coordinate system. The girder moves along the X_c axis. The trolley moves along the Y_c axis. Coordinates of the trolley and load are shown as the figure. θ is the swing angle of the load in a space and is subcategorized into two components: θ_x and θ_y. l is the rope length from the trolley to the load.

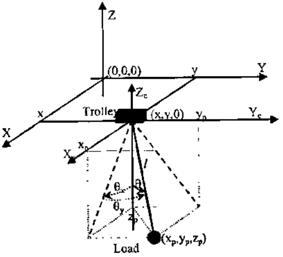

Figure 1. *The description of the 3-D crane*

The position of load (x_p, y_p, z_p) in fixed coordinate can be performed:

$$x_p = x + l\sin\theta_x\cos\theta_y;$$
$$y_p = y + l\sin\theta_y; \quad (1)$$
$$z_p = -l\cos\theta_x\cos\theta_y$$

This study refers to three simultaneous movement of girder, trolley and load. Therefore, the parameters x, y, l, θ_x and θ_y is defined in the general coordinates to describe motion of overhead crane.

The motion of 3-D overhead crane is based on Lagrange's equation. Here the load is assumed as a point mass located at the center. The mass and the springiness of the rope are ignored. T is called the kinetic energy of cranes including the girder, the trolley and the load; P is called the potential energy of the crane.

$$T = T_{girder} + T_{trolley} + T_{load}$$
$$= \frac{1}{2}\left(M_x\dot{x}^2 + M_y\dot{y}^2 + M_l\dot{l}^2\right) + \frac{m}{2}v_p^2 \quad (2)$$
$$P = P_{taitrong} = mgl\left(1 - \cos\theta_x\cos\theta_y\right) \quad (3)$$

where M_x is a traveling component of the crane system mass, M_y is a traversing component and M_l is a hoisting component m, g and v_p are the load mass, the gravity and the load velocity, respectively.

$$v_p^2 = \dot{x}_p^2 + \dot{y}_p^2 + \dot{z}_p^2$$
$$v_p^2 = \dot{x}_p^2 + \dot{y}_p^2 + \dot{z}_p^2 = \dot{x}^2 + \dot{y}^2 + \dot{l}^2 +$$
$$l^2\cos^2\theta_y\dot{\theta}_x^2 + l^2\dot{\theta}_y^2 + 2(\sin\theta_x\cos\theta_y\dot{l} +$$
$$l\cos\theta_x\cos\theta_y\dot{\theta}_x - l\sin\theta_x\sin\theta_y\dot{\theta}_y)\dot{x} + \quad (4)$$
$$2(\sin\theta_y\dot{l} + l\cos\theta_y\dot{\theta}_y)\dot{y}$$

The Lagrange function is defined as:

$$L = T - P = \frac{1}{2}\left(M_x\dot{x}^2 + M_y\dot{y}^2 + M_l\dot{l}^2\right)$$
$$+\frac{m}{2}v_p^2 + mgl\left(\cos\theta_x\cos\theta_y - 1\right) \quad (5)$$

The dissipation function (mainly due to friction) is defined as follows:

$$\Phi = \frac{1}{2}(D_x\dot{x}^2 + D_y\dot{y}^2 + D_l\dot{l}^2) \quad (6)$$

where D_x, D_y và D_l denote the viscous damping coefficients according to the *x*, *y* and *l* motion.

The general Lagrange equations is written:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = -\frac{\partial P}{\partial q_i} - \frac{\partial \Phi}{\partial q_i} + F_{q_i} \ (i = 1 \div 5) \quad (7)$$

where F_{q_i} is the corresponding generalized force *ith*, which belongs to the generalized coordinate system. The equations of motion of the crane system are defined by inserting L and Φ in Lagrange equations with the generalized coordinate system *x*, *y*, *l*, θ_x, θ_y:

$$(M_x + m)\ddot{x} + ml\cos\theta_x\cos\theta_y\ddot{\theta}_x - ml\sin\theta_x\sin\theta_y\ddot{\theta}_y$$
$$+ m\sin\theta_x\cos\theta_y\ddot{l} + D_x\dot{x} + 2m\cos\theta_x\cos\theta_y\dot{l}\dot{\theta}_x \quad (8)$$
$$-2m\sin\theta_x\sin\theta_y\dot{l}\dot{\theta}_y - ml\sin\theta_x\cos\theta_y\theta_x^2$$
$$-2ml\cos\theta_x\sin\theta_y\dot{\theta}_x\dot{\theta}_y - ml\sin\theta_x\cos\theta_y\theta_y^2 = f_x$$

$$(M_y + m)\ddot{y} + ml\cos\theta_y\ddot{\theta}_y + m\sin\theta_y\ddot{l} \quad (9)$$
$$+ D_y\dot{y} + 2m\cos\theta_y\dot{l}\dot{\theta}_y - ml\sin\theta_y\dot{\theta}_y^2 = f_y$$

$$(M_l+m)\ddot{l}+m\sin\theta_x\cos\theta_y\ddot{x}+m\sin\theta_y\ddot{y}+D_l\dot{l}$$
$$-ml\cos^2\theta_y\dot{\theta}_x^2-ml\dot{\theta}_y^2-mg\cos\theta_x\cos\theta_y=f_l \quad (10)$$

$$ml^2\cos^2\theta_y\ddot{\theta}_x+ml\cos\theta_x\cos\theta_y\ddot{x}$$
$$+2ml\cos^2\theta_y\dot{l}\dot{\theta}_x-2ml^2\sin\theta_y\cos\theta_y\dot{\theta}_x\dot{\theta}_y \quad (11)$$
$$+mgl\sin\theta_x\cos\theta_y=0$$

$$ml^2\ddot{\theta}_y+ml\cos\theta_y\ddot{y}-ml\sin\theta_x\sin\theta_y\ddot{x}$$
$$+2ml\dot{l}\dot{\theta}_y+ml^2\cos\theta_y\sin\theta_y\dot{\theta}_x^2 \quad (12)$$
$$+mgl\cos\theta_x\sin\theta_y=0$$

where f_x, f_y, f_l are the driving force of the girders, the trolley and the load for the *x, y, l* motions, respectively.

The dynamic model of crane is equivalent to the dynamic model of robot having three soft bindings. The dynamic model (8) - (12) can be performed in the form of the matrix vector, as follows:

$$M(q)\ddot{q}+D\dot{q}+C(\dot{q},q)\dot{q}+G(q)=F \quad (13)$$

where q is the state vector, F is the driving force vector, $G(q)$ is gravitational vector and D is dissipation matrix because of the friction, respectively:

$$q=[x,\ y,\ l,\ \theta_x,\ \theta_y]^T$$
$$F=[f_x,\ f_y,\ f_l,\ 0\ ,\ 0]^T$$
$$G(q)=(0,0,-mg\cos\theta_x\cos\theta_y,mgl\sin\theta_x\cos\theta_y,$$
$$mgl\cos\theta_x\sin\theta_y)^T$$
$$D=diag(D_x,D_y,D_l,0,0)$$

The symmetric mass matrix $M(q)\in R^{(5\times5)}$ is denoted:

$$M(q)=\begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} & m_{15}\\ 0 & m_{22} & m_{23} & 0 & m_{25}\\ m_{31} & m_{32} & m_{33} & 0 & 0\\ m_{41} & 0 & 0 & m_{44} & 0\\ m_{51} & m_{52} & 0 & 0 & m_{55}\end{bmatrix}$$

$$m_{11}=M_x+m;m_{13}=m\sin\theta_x\cos\theta_y;$$
$$m_{14}=ml\cos\theta_x\cos\theta_y;m_{15}=-ml\sin\theta_x\sin\theta_y;$$
$$m_{22}=M_y+m;m_{23}=m\sin\theta_y;$$
$$m_{25}=ml\cos\theta_y;m_{31}=m\sin\theta_x\cos\theta_y;$$
$$m_{32}=m\sin\theta_y;m_{33}=M_l+m;$$
$$m_{41}=ml\cos\theta_x\cos\theta_y;m_{44}=ml^2\cos^2\theta_y;$$
$$m_{51}=-ml\sin\theta_x\sin\theta_y;m_{52}=ml\cos\theta_y;m_{55}=ml^2$$

M(q) is positive definite when $l>0$ and $|\theta_y|<\pi/2$. $C(\dot{q},q)\in R^{5\times5}$ is the matrix of centrifugal force and Coriolis.

$$C(\dot{q},q)=\begin{bmatrix} 0 & 0 & c_{13} & c_{14} & c_{15}\\ 0 & 0 & c_{23} & 0 & c_{25}\\ 0 & 0 & 0 & c_{34} & c_{35}\\ 0 & 0 & c_{43} & c_{44} & c_{45}\\ 0 & 0 & c_{53} & c_{54} & c_{55}\end{bmatrix}$$

$$c_{13}=m\cos\theta_x\cos\theta_y\dot{\theta}_x-m\sin\theta_x\sin\theta_y\dot{\theta}_y;$$
$$c_{14}=ml\cos\theta_x\cos\theta_y-ml\sin\theta_x\cos\theta_y\dot{\theta}_x-ml\cos\theta_x\sin\theta_y\dot{\theta}_y,$$
$$c_{15}=-m\sin\theta_x\sin\theta_y\dot{l}-ml\cos\theta_x\sin\theta_y\dot{\theta}_x-ml\sin\theta_x\cos\theta_y\dot{\theta}_y;$$
$$c_{23}=m\cos\theta_y\dot{\theta}_y;c_{25}=m\cos\theta_y\dot{l}-ml\sin\theta_y\theta_y;$$
$$c_{34}=-ml\cos^2\theta_y\dot{\theta}_x;c_{35}=-ml\dot{\theta}_y;c_{43}=ml\cos^1\theta_y\dot{\theta}_x;$$
$$c_{44}=ml\cos^2\theta_y\dot{l}-ml^2\sin\theta_y\cos\theta_y\theta_y;$$
$$c_{45}=-ml^2\sin\theta_y\cos\theta_y\dot{\theta}_x;c_{53}=ml\dot{\theta}_y;$$
$$c_{54}=ml^2\cos\theta_y\sin\theta_y\dot{\theta}_x;c_{55}=ml\dot{l};$$

SIMULATION OF CRANE SYSTEM RESPONSE WITH VARIABLE PARAME-TERS

In this section, the dynamic of 3-D crane (13) will be analyzed in the time domain and frequency domain. The values of the nominal parameters are determined by crane models in the laboratory:

$$M_x=12.85kg;D_x=30Ns/m;M_y=5.85kg;$$
$$D_y=20Ns/m;M_l=2.85kg;D_l=50Ns/m;$$
$$m=0.85kg;f_x=60N;f_y=30N;$$
$$f_l=-8N;l>0$$

The gravity acceleration is $g=9.8m/s^2$. Simulation time is 10s, the sampling time is 1ms. The position and swing angle responses of the system and the power spectral density are analyzed and evaluated.

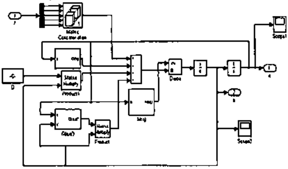

Figure 2. *General schematic simulation*

The system response with different loads

To observe the affects of the payload on the system dynamic, various payloads are simulated. The results showed most clearly when the mass of load changes from 0,85kg to 5,50kg. Figure 3 shows the position responses in the x, y, z axis. There are no large oscillation in the position response . Table 1 synthesizes the relation between the mass of load and the trolley positions. Respectively, figures 4 and 5 indicated responses of swing angle in the x and y directions when the mass of the load is changed. This relationship has also been summarized as in the Table 1.

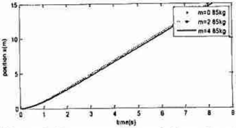

Figure 3. *Position response in the x directions with variation of payload*

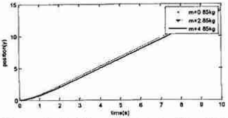

Figure 4. *Position response in the y directions with variation of payload*

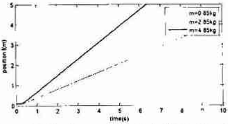

Figure 5. *Position response in the z directions with variation of payload*

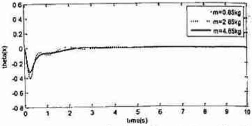

Figure 6. *Swing angle θ_x with variation of payload*

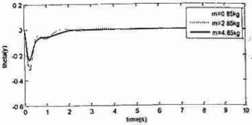

Figure 7. *Swing angle θ_y with variation of payload*

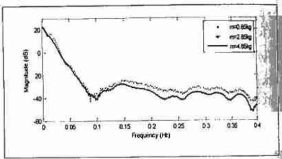

Figure 8. *Power spectral density of θ_x with variation of payload*

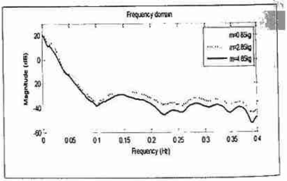

Figure 9. *Power spectral density of θ_y with variation of payload*

Table 1. *The relation between variation of payload with trolley position and swing angles*

Payload (kg)	Trolley position (m) (average)			Swing angles (max-min)	
	x direction	y direction	z direction	θ_x (rad)	θ_y (rad)
m=0.85	5.863	4.533	0.1351	±0.6626	±0.5112
m=1.50	5.797	4.456	0.5295	±0.5336	±0.4196
m=2.85	5.670	4.310	1.3700	±0.4076	±0.3150
m=3.50	5.611	4.243	1.7620	±0.3724	±0.2839
m=4.85	5.491	4.108	2.3270	±0.3219	±0.2383
m=5.50	5.435	4.045	2.5120	±0.3038	±0.2218

The findings show that if the mass of load is increased, the swing angle will decrease, vibration frequency will also decrease, oscillation period will be shorter. Figure 7 and Figure 8 shows the power spectral density corresponding to the swing angle in the x direction and the y direction. It proves that the resonance with oscillation frequency increases when the load increases. Thus, this study shows that in order to reduce the vibrations of the system, we can limit the range of the load mass. Accordingly, this range is called "effective parameter range,,. Even then, if the system is not yet equipped with modern controllers, high performance with "effective parameter range,, is maintained. In this case, when the load mass is within 4kg to 5kg. Swing angle and also frequency reduces, the settling time is less than 3 seconds.

The system response with different hoisting force

To observe more clearly the effects of the system parameters to the vibration of the load, especially hoisting force, here we consider f_l = [-20N, 20N]. Girder force, trolley force and other parameters are constant.

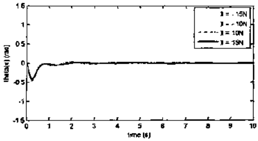

Figure 10. *Swing angle θ_x with variation of hoisting force*

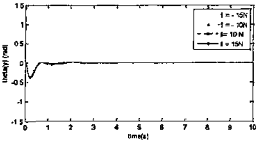

Figure 11. *Swing angle θ_y with variation of hoisting force*

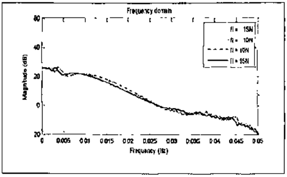

Figure 12. *Power spectral densitys of swing angle θ_x with variation of hoisting force*

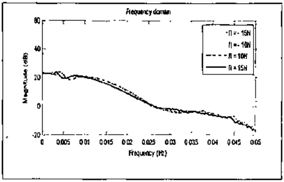

Figure 13. *The power spectral density of swing angle θ_x with variation of hoisting force*

Table 2. *Relation between hoisting force with swing angles*

Hoisting force	Swing angle (max-min)	
(N)	θ_x (rad)	θ_y (rad)
f_l = -15	±1.271	±1.251
f_l = -10	±0.7208	±0 5383
f_l = -5	±0.6291	±0 4946
f_l = 5	±0.5041	±0.4245
f_l = 10	±0.4598	±0.3956
f_l = 15	±0 4234	±0.3707

Figure 8 and Figure 9 show that the swing angles as lifting loads are less oscillator than as lowering loads. The vibration of the response is proportional to the lowering force and inversely proportional to the lifting force. Figure 10, Figure 11 described power spectral densitys of swing angles. Oscillation frequency is also proportional to the lowering force and inversely proportional to the lifting force. Statistical parameters in Table 2 shows the relation between the hoisting force with the swing angle. Such the results also showed that if the lifting force is from 10N to 15N, the quality of system is good, the settling time is less than 1 second, the overshoot is about 12%, oscillation frequency is also smaller. The results confirmed that it is not neccessary to design a new controller if the hoisting force is varied within the "effective parameter range,,.

CONCLUSION

This study presents the development of a dynamics model of a 3-D overhead crane base on the Euler-Lagrange approach. The model was simulated with bang – bang force input. The trolley position and the swing angle response have been described and analyzed in the time domain and frequency domain. The affection of mass load, hoisting force to the dynamic characteristic of the system are also analyzed also discussed. These results are very useful and important to develop effective control methods and control algorithms for the system 3-D crane with different loads and driving forces.

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TÓM TẮT
MÔ HÌNH HÓA VÀ PHÂN TÍCH ĐỘNG HỌC CỦA HỆ THỐNG CẦU TRỤC 3D KHI THAY ĐỔI LỰC NÂNG HẠ VÀ KHỐI LƯỢNG TẢI TRỌNG

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Cầu trục được sử dụng rất phổ biến trong công nghiệp, trong quân sự để di chuyển những trọng tải nặng, hoặc lắp ghép những cấu kiện lớn. Ba chuyển động cơ bản của cầu trục là hành trình học, hành trình ngang và nâng hạ tải trọng. Sự rung lắc của tải trọng khi di chuyển đe dọa đến vấn đề an toàn và ảnh hưởng đến hiệu quả làm việc. Tăng tốc độ làm việc nhằm nâng cao hiệu suất càng gây ra sự rung lắc làm hao tổn thời gian, dẫn đến không đạt kết quả mong muốn. Bài viết này phân tích và đề xuất giải pháp nâng cao hiệu quả khi cho cầu trục làm việc trong điều kiện tham số thích hợp. Bài viết đồng thời mô tả mô hình động lực học của hệ thống cầu trục trong không gian ba chiều dựa vào phương pháp Euler- Lagrange, gồm mô tả những chuyển động của tải trong theo hướng dọc, ngang và nâng hạ. Những ảnh hưởng của sự thay đổi khối lượng tải trọng và lực kéo nâng hạ đến đáp ứng hệ thống trên miền thời gian và miền tần số được phân tích qua kết quả mô phỏng. Bài báo cũng đề xuất vùng tham số làm việc hiệu quả. Cuối cùng là một số kết luận.

Từ khóa: *Mô hình động học; cầu trục 3-D; phương pháp Euler- Lagrange; miền thời gian và miền tần số; mật độ phổ công suất, vùng tham số hiệu quả*

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