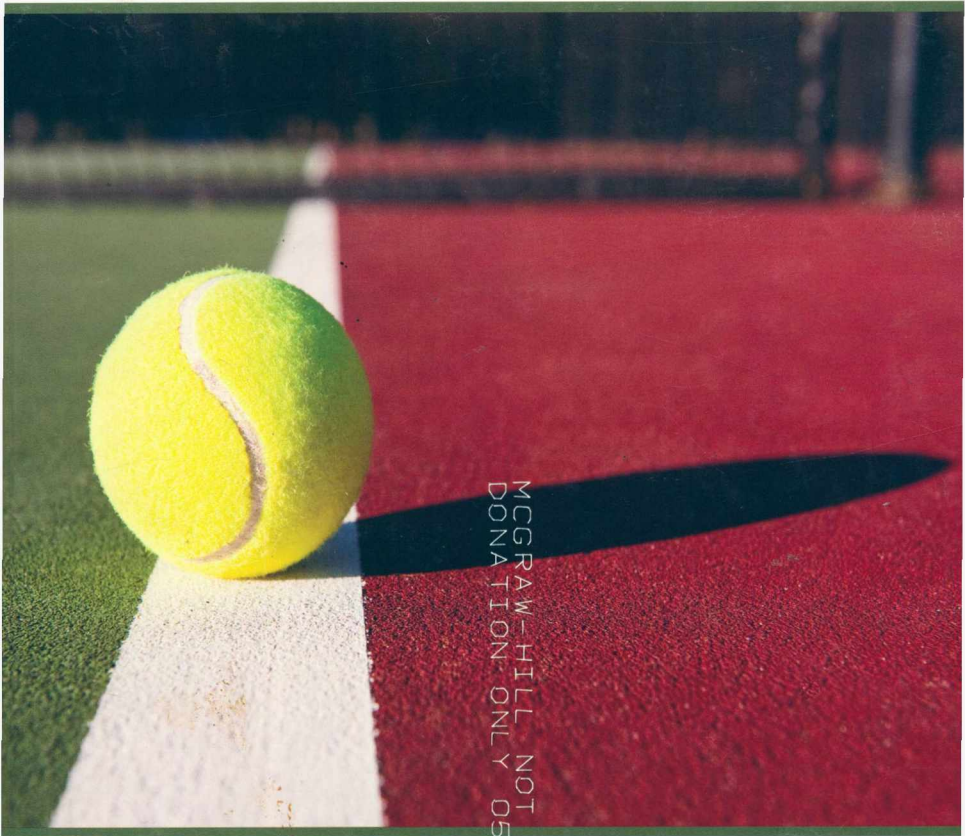




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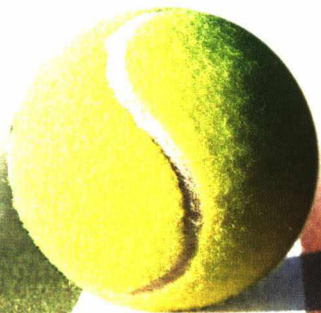


# College Algebra

## ESSENTIALS

# College Algebra

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**Julie Miller**  
*Daytona State College*

Digital Author  
**Donna Gerken**  
*Miami Dade College*





COLLEGE ALGEBRA ESSENTIALS

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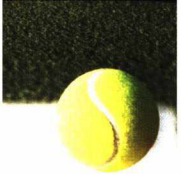
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# About the Authors



**Julie Miller** is from Daytona State College where she has taught developmental and upper-level mathematics courses for 20 years. Prior to her work at DSC, she worked as a software engineer for General Electric in the area of flight and radar simulation. Julie earned a bachelor of science in applied mathematics from Union College in Schenectady, New York, and a master of science in mathematics from the University of Florida. In addition to this textbook, she has authored eight textbooks in developmental mathematics, several course supplements for trigonometry and precalculus, as well as several short works of fiction and nonfiction for young readers.

“My father is a medical researcher, and I got hooked on math and science when I was young and would visit his laboratory. I remember doing simple calculations with him and using graph paper to plot data points for his experiments. He would then tell me what the peaks and features in the graph meant in the context of his experiment. I think that applications and hands-on experience made math come alive for me, and I’d like to see math come alive for my students.”

**Donna Gerken** is currently a professor at Miami Dade College where she has taught developmental courses, honors classes, and upper-level mathematics classes for 30 years. Throughout the years, she has been involved with many projects at Miami Dade on curriculum redesign and the use of technology in the classroom. Donna’s bachelor of science in mathematics and master of science in mathematics are both from the University of Miami. Before finishing her undergraduate and graduate degrees at the University of Miami, she also attended Miami Dade College as a student where she discovered an amazing group of faculty who inspired her to continue on with a career in mathematics. Donna has kept a quote for many years that was passed along from one of her undergraduate professors:

*Pure mathematics is, in its way, the poetry of logical ideas.*

—Albert Einstein, in a letter to the editor of the New York Times upon the death of Emmy Noether.

If Donna is not in the classroom or peering into her computer screen, she can be found reading, in her kitchen cooking for crowds, or working out with friends at the gym.

## Letter from the Authors

For many students, college algebra is a daunting course that serves as a gateway between developmental math and the realm of higher level mathematics taken by engineers and scientists. For this reason many years ago, we began writing a series of textbooks to bridge the gap between preparatory courses and the more abstract world of college algebra. For thousands of students, the Miller/O’Neill/Hyde textbook series has provided a solid foundation in intermediate algebra. Now, we want to address student needs on the other side of the bridge. Our goal is to carry the clear, concise writing style and popular pedagogical features of our textbooks to college algebra students.

The main objectives of this college algebra textbook are threefold:

- To provide students with a clear and logical presentation of the basic concepts that will prepare them for continued study in mathematics.
- To help students develop logical thinking and problem-solving skills that will benefit them in all aspects of life.
- To motivate students by demonstrating the significance of mathematics in their lives through practical applications.



*Julie Miller*

julie.miller.math@gmail.com

*Donna Gerken*

dgerken@mdc.edu



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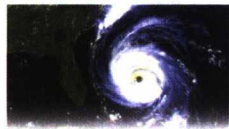


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# Key Features



## Clear, Precise Writing

Because a diverse group of students take this course, Julie Miller has written this manuscript to use simple and accessible language. Through her friendly and engaging writing style, students are able to understand the material easily.

## Exercise Sets

The exercises at the end of each section are graded, varied, and carefully organized to maximize student learning:

- **Review Exercises** begin the section-level exercises and ensure that students have the prerequisite skills to complete the homework sets successfully.
- **Concept Connections** exercises prompt students to review the vocabulary and key concepts presented in the section.
- The core exercises are presented next and are grouped by objective. These exercises are linked to examples in the text and direct students to similar problems whose solutions have been stepped-out in detail.
- **Mixed Exercises** do *not* refer to specific examples so that students can dip into their mathematical toolkit and decide on the best technique to use.
- **Write About It** exercises are designed to emphasize mathematical language by asking students to explain important concepts.
- **Technology Connections** exercises require the use of a graphing utility and are found at the end of exercise sets. They can be easily skipped for those who do not encourage the use of calculators.

**SECTION 4.3 Practice Exercises**

**Review Exercises**  
For Exercises 1–2, write an equation for the inverse function.  
1.  $f(x) = 3x + 4$       2.  $f(x) = 2x - 1$

For Exercises 3–6, fill in the blank to make a true statement.  
3.  $f(x) = 4x + 1$       4.  $2 = 5x$       5.  $f(x) = \frac{1}{2}x + 3$       6.  $f(x) = \frac{1}{3}x + 4$

**Concept Connections**  
7. State whether  $f(x) = 2x^2 + 3x - 1$  and  $f^{-1}(x) = \frac{1}{2}x + 1$  are inverse functions. Justify your answer.  
8.  $f(x) = \log_2(x)$  is called the logarithmic function. State its domain, range, and  $f^{-1}(x)$ .  
9.  $f(x) = \log_2(x)$  is called the logarithmic function. State its domain, range, and  $f^{-1}(x)$ .  
10. The inverse of an exponential function  $f(x) = a^x$  is called the logarithmic function  $f^{-1}(x)$ .  
11. The logarithmic function  $f(x) = \log_2(x)$  is called the logarithmic function. State its domain, range, and  $f^{-1}(x)$ .  
12. Exponential  $f(x) = 2^x$  and logarithmic  $f(x) = \log_2(x)$  are inverse functions. State their domains and ranges.

**Mixed Exercises**  
For Exercises 85–92, find an equation for the inverse function.  
85.  $f(x) = 2x + 3$       86.  $f(x) = 3x - 5$       87.  $f(x) = 4x + 1$       88.  $f(x) = 5x - 2$   
89.  $f(x) = 6x + 7$       90.  $f(x) = 7x + 8$       91.  $f(x) = 8x + 9$       92.  $f(x) = 9x + 10$

**Write About It**  
113. Explain the equivalence property of exponential equations.  
114. Explain the process to solve the equation  $5^x = 125$ .

**Expanding Your Skills**  
For Exercises 115–126, solve the equation.  
115.  $10^x = 10^{x+1} - 9$       116.  $2^x = 2^{x+1} - 4$       117.  $3^x = 3^{x+1} - 8$   
118.  $4^x = 4^{x+1} - 12$       119.  $5^x = 5^{x+1} - 20$       120.  $6^x = 6^{x+1} - 30$   
121.  $7^x = 7^{x+1} - 42$       122.  $8^x = 8^{x+1} - 56$       123.  $9^x = 9^{x+1} - 72$       124.  $10^x = 10^{x+1} - 90$

**Technology Connections**  
For Exercises 127–130, an equation is given in the form  $Y_1(x) = Y_2(x)$ . Graph  $Y_1$  and  $Y_2$  on a graphing utility on the window  $[-10, 10]$  by  $[-10, 10]$ . Then approximate the points of intersection to approximate the solutions to the equation. Round to 4 decimal places.  
127.  $1.1^x = 1.1^{x+1} - 10$       128.  $1.2^x = 1.2^{x+1} - 10$       129.  $1.3^x = 1.3^{x+1} - 10$       130.  $1.4^x = 1.4^{x+1} - 10$

**PROBLEM RECOGNITION EXERCISES**

**Analyzing Functions**  
For Exercises 1–14:

a. Write the domain.	b. Write the range.	c. Find the x-intercepts.	d. Find the y-intercept.
e. Determine the asymptotes of applicability.	f. Determine the intervals over which the function is increasing.		

1.  $f(x) = \frac{1}{2}x + 3$       2.  $f(x) = 2x - 1$       3.  $f(x) = \frac{1}{3}x + 4$       4.  $f(x) = \frac{1}{4}x + 5$

5.  $f(x) = \frac{1}{5}x + 6$       6.  $f(x) = \frac{1}{6}x + 7$       7.  $f(x) = \frac{1}{7}x + 8$       8.  $f(x) = \frac{1}{8}x + 9$

9.  $f(x) = \frac{1}{9}x + 10$       10.  $f(x) = \frac{1}{10}x + 11$       11.  $f(x) = \frac{1}{11}x + 12$       12.  $f(x) = \frac{1}{12}x + 13$

13.  $f(x) = \frac{1}{13}x + 14$       14.  $f(x) = \frac{1}{14}x + 15$       15.  $f(x) = \frac{1}{15}x + 16$       16.  $f(x) = \frac{1}{16}x + 17$

A.      B.      C.      D.

E.      F.      G.      H.

## Problem Recognition Exercises

**Problem Recognition Exercises** appear in strategic locations in each chapter of the text. These exercises help students compare and contrast a variety of problem types and determine which mathematical tool to apply to a given problem.



## Examples

- The examples in the textbook are stepped-out in detail with thorough annotations at the right explaining each step.
- Following each example is a similar **Skill Practice** exercise to engage students by practicing what they have just learned.
- For the instructor, references to an even-numbered exercise are provided next to each example. These exercises are highlighted in the exercise sets and mirror the related examples. With increased demands on faculty time, this has been a popular feature to help faculty write their lectures and develop their presentation of material. If an instructor presents all of the highlighted exercises, then each objective of that section of text will be covered.

## Modeling and Applications

One of the most important tools to motivate our students is to make the mathematics they learn meaningful in their lives. The textbook is filled with robust applications and numerous opportunities for mathematical modeling for those instructors looking to incorporate these features into their course.

## Callouts

Throughout the text, popular tools are included to highlight important ideas. These consist of

- Tip** boxes that offer additional insight to a concept or procedure.
- Avoiding Mistakes** boxes that fend off common mistakes.
- Point of Interest** boxes that offer interesting and historical mathematical facts.
- Instructor Notes** to assist with lecture preparation.

### EXAMPLE 6 Using Logistic Growth to Model Population

The population of California,  $P(t)$  (in millions) can be approximated by the logistic growth function

$$P(t) = \frac{95.2}{1 + 1.8e^{-0.013t}}$$

where  $t$  is the number of years since the year 2000.

a. Determine the population in the year 2000.

b. Use this function to determine the time required for the population of California to double from its value in 2000. Compare this with the result from Example 5(c).

c. What is the limiting value of the population of California under this model?

**Solution:**

a.  $P(t) = \frac{95.2}{1 + 1.8e^{-0.013t}}$

$$P(0) = \frac{95.2}{1 + 1.8e^{-0.013(0)}} = \frac{95.2}{1 + 1.8(1)} = 34$$

The population was approximately 34 million in the year 2000.

b.  $P(t) = \frac{95.2}{1 + 1.8e^{-0.013t}}$

$$95.2 = \frac{95.2}{1 + 1.8e^{-0.013t}} \cdot (1 + 1.8e^{-0.013t})$$

$$95.2(1 + 1.8e^{-0.013t}) = 95.2$$

$$95.2 + 1.7136e^{-0.013t} = 95.2$$

$$1.7136e^{-0.013t} = 0$$

$$e^{-0.013t} = \frac{0}{1.7136} = 0$$

$$-0.013t = \ln(0)$$

$$t = \frac{\ln(0)}{-0.013}$$

The population will double in approximately 87.6 yr. This is 9 yr later than the predicted value from Example 5(c).

The graphs of  $P(t) = \frac{95.2}{1 + 1.8e^{-0.013t}}$  and  $P(t) = 34e^{0.013t}$  are shown in Figure 4.17. Notice that the two models agree relatively closely for short-term population growth (out to about 2000). However, in the long term, the unbounded exponential model breaks down. The logistic growth model approaches a limiting population, which is reasonable due to the limited resources to sustain a large human population.

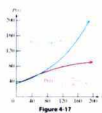


Figure 4.17

c.  $P(t) = \frac{95.2}{1 + 1.8e^{-0.013t}}$

$$P(t) = \frac{95.2}{1 + 1.8e^{-0.013t}}$$

As  $t$  becomes large, the denominator of  $\frac{95.2}{1 + 1.8e^{-0.013t}}$  becomes large. This causes the quotient to approach zero. Therefore, as  $t$  approaches infinity,  $P(t)$  approaches 95.2. Under this model, the limiting value for the population of California is 95.2 million.

Figure 4.17: A graph showing two population models over time. The x-axis is time t in years since 2000, ranging from 0 to 200. The y-axis is population P(t) in millions, ranging from 0 to 100. One curve is a blue exponential growth function P(t) = 34e^{0.013t}, which rises steeply. The other curve is a red logistic growth function P(t) = 95.2 / (1 + 1.8e^{-0.013t}), which starts at (0, 34) and levels off towards a horizontal asymptote at P = 95.2. The two curves are nearly identical until about t = 2000.

The intermediate value theorem can be used repeatedly in a technique called the bisection method to approximate the value of a zero. See the online group activity "Investigating the Bisection Method for Finding Zeros."

### Point of Interest

The modern definition of a computer is a programmable device designed to carry out a sequence of arithmetic or logical operations. However, the word "computer" originally referred to a person who did such calculations using paper and pencil. "Human computer" was notably used in the aerospace industry to predict the path of ballistic missiles and to produce astronomical tables critical to surveying and navigation. Later, during World War II, and human computers developed ballistic firing tables that would describe the trajectory of a shell.

Computing tables of values was very time consuming, and the "computer" would often interpolate to find intermediate values within a table. Interpolation is a method by which intermediate values between two numbers are estimated. Often the interpolated values were based on a polynomial function.



### 4. Sketch a Polynomial Function

The graph of a polynomial function may also have "turning points." For example, consider  $f(x) = x^3 + 2x^2 - 2x - 2$ . See Figure 3.11. Multiplying the leading terms within the factors, we have a leading term of  $10x^2(1 - x^2)$ . Therefore, the end behavior of the graph is up to the left and up to the right.

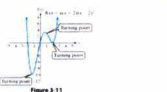


Figure 3.11

Starting from the far left, the graph of  $f$  decreases to the  $x$ -intercept of  $-2$ . Since  $-2$  is a zero with an odd multiplicity, the graph must cross the  $x$ -axis at  $-2$ . For the same reason, the graph must cross the  $x$ -axis again at the origin. Therefore, somewhere between  $-2$  and  $0$ , the graph must "turn around." This point is called a "turning point."

The turning points of a polynomial function are the points where the function changes from increasing to decreasing or vice versa.

### Number of Turning Points of a Polynomial Function

Let  $f$  represent a polynomial function of degree  $n$ . Then the graph of  $f$  has at most  $n - 1$  turning points.

At this point we are ready to outline a strategy for sketching a polynomial function.

### TIP

Even with advanced techniques, calculating the size of a graphing utility, it is often difficult or impossible to find the exact location of the turning points of a polynomial function.

### Avoiding Mistakes

A polynomial of degree  $n$  may have fewer than  $n - 1$  turning points. For example,  $f(x) = x^3$  is a degree 3 polynomial function. Concluding that it could have a maximum of two turning points, yet the graph has no turning points.



**TECHNOLOGY CONNECTIONS**

**Graphing an Exponential Function**

A graphing utility can be used to graph and analyze exponential functions. The table shows several values of  $f(x) = (1/2)^{x-1}$  for selected values of  $x$ .

x	f(x)
1	1
2	0.5
3	0.25
4	0.125
5	0.0625
6	0.03125
7	0.015625
8	0.0078125
9	0.00390625
10	0.001953125

The graph of  $f(x) = (1/2)^{x-1}$  is shown on the viewing window  $[-0, 10.000]$ ,  $[0.000]$  by  $[-1, 0.2, 1.2, 0.1]$ . Notice that the graph is a decreasing exponential function because the base is between 0 and 1.

## Graphing Calculator Coverage

Material is presented throughout the book illustrating how a graphing utility can be used to view a concept in a graphical manner. The goal of the calculator material is not to replace algebraic analysis, but rather to enhance understanding with a visual approach. Graphing calculator examples are placed in self-contained boxes and may be skipped by instructors who choose not to implement the calculator. Similarly, the graphing calculator exercises are found at the end of the exercise sets and may also be easily skipped.

## End-of-Chapter Materials

The textbook has the following end-of-chapter materials for students to review before test time.

- Brief summary with references to key concepts. A detailed summary is located at [www.mhhe.com/millerca](http://www.mhhe.com/millerca).
- Chapter review exercises.
- Chapter test.
- Cumulative review exercises. These exercises cover concepts in the current chapter as well as all preceding chapters.

**Interpreting a Quadratic Model**

Time (sec)	Height (ft)
1	240
3	624
4.8	860.2
7.4	1018.2
9	1008
10.6	915.8
11.8	748.8

Height of Rocket vs. Time

## Digital Media

Digital assets were created exclusively by the author team to ensure that the author voice is present and consistent throughout the supplement package.

- The **digital coauthor**, Donna Gerken, ensures that each algorithm in the online homework has a stepped-out solution unique to the Miller style.
- Julie Miller created **video content** (lecture videos, exercise videos, graphing calculator videos, and Excel videos) to give students access to classroom-type instruction by the author.
- Julie Miller constructed over **50 dynamic math animations** to accompany the college algebra text. The animations are diverse in scope and give students an interactive approach to conceptual learning. The animated content illustrates difficult concepts by leveraging the use of on-screen movement where static images in the text may fall short. They are organized in Connect Math Hosted by ALEKS by chapter and section, as well as grouped by various categories including a Functions Library, Applications/Modeling, and Graphing.
- The authors developed lecture notes in both ready-made PDF format and in Word format so that instructors can tailor the material to their course.
- The authors created a library of activities in the Student Resource Manual that include group activities and Wolfram Alpha activities.

Graph:  $y = \log_3(x-1) - 3$

So instead of being the line  $x = 0$ , we have the line  $x = 1$ .