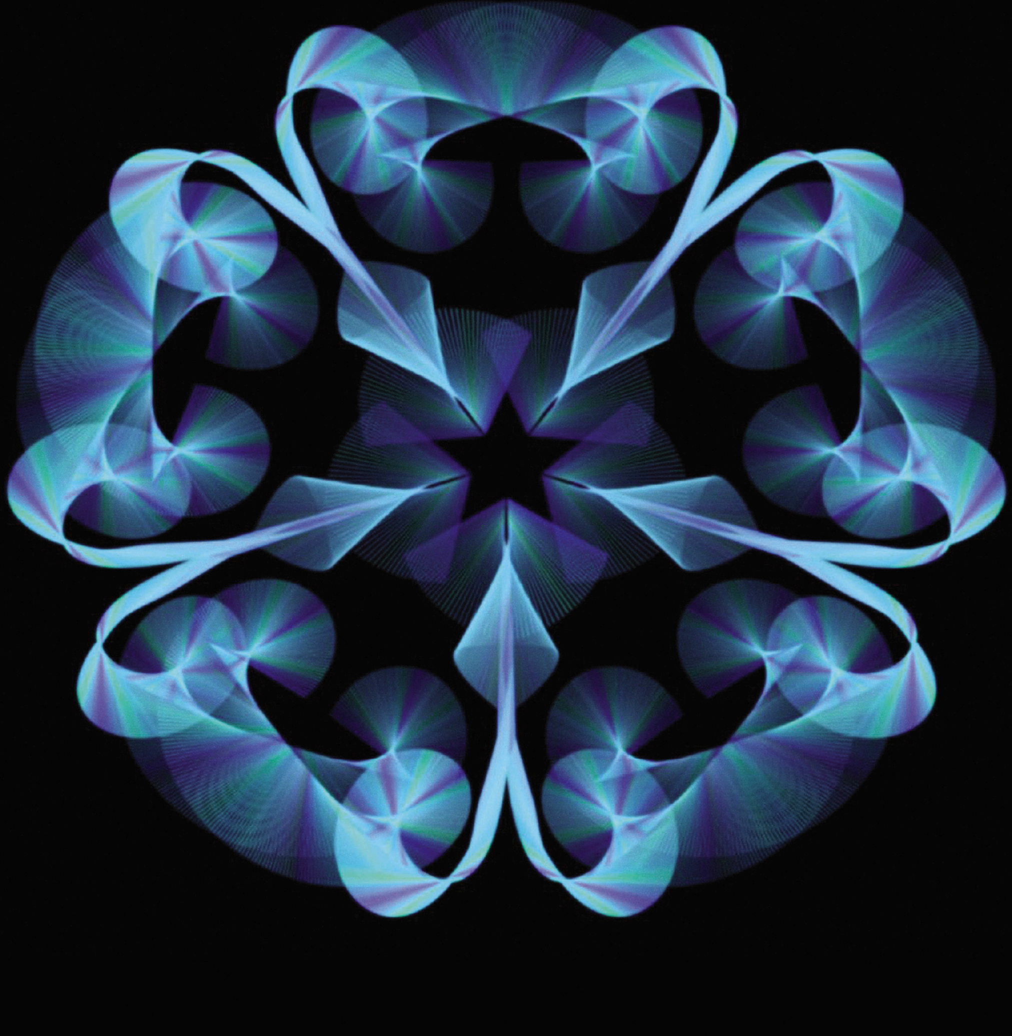
Joseph A. Gallian

CONTEMPORARY ABSTRACT ALGEBRA

Ninth Edition



Notations

(The number after the item indicates the page where the notation is defined.)

SET THEORY

$\cap_{i \in I} S_i$	intersection of sets S_i, $i \in I$
$\cup_{i \in I} S_i$	union of sets S_i, $i \in I$
$[a]$	$\{x \in S \mid x \sim a\}$, equivalence class of S containing a, 18
$\lvert s \rvert$	number of elements in the set of S

SPECIAL SETS

Z	integers, additive groups of integers, ring of integers
Q	rational numbers, field of rational numbers
Q^+	multiplicative group of positive rational numbers
F^*	set of nonzero elements of F
$\mathbf{R}$	real numbers, field of real numbers
$\mathbf{R}^+$	multiplicative group of positive real numbers
$\mathbf{C}$	complex numbers

FUNCTIONS AND ARITHMETIC

f^{-1}	inverse of the function f
$t \mid s$	t divides s, 3
$t \nmid s$	t does not divide s, 3
$\gcd(a, b)$	greatest common divisor of the integers a and b, 4
$\text{lcm}(a, b)$	least common multiple of the integers a and b, 6
$\lvert a + b \rvert$	$\sqrt{a^2 + b^2}$, 13
$\phi(a)$	image of a under ϕ, 20
$\phi: A \to B$	mapping of A to B, 21
gf, $\alpha\beta$	composite function, 21

ALGEBRAIC SYSTEMS

D_4	group of symmetries of a square, dihedral group of order 8, 33
D_n	dihedral group of order $2n$, 34
e	identity element, 43
Z_n	group $\{0, 1, \ldots, n - 1\}$ under addition modulo n, 44
$\det A$	the determinant of A, 45
$U(n)$	group of units modulo n (that is, the set of integers less than n and relatively prime to n under multiplication modulo n), 46
$\mathbf{R}^n$	$\{(a_1, a_2, \ldots, a_n) \mid a_1, a_2, \ldots, a_n \in \mathbf{R}\}$, 47
$SL(2, F)$	group of 2×2 matrices over F with determinant 1, 47
$GL(2, F)$	2×2 matrices of nonzero determinants with coefficients from the field F (the general linear group), 48
g^{-1}	multiplicative inverse of g, 51
$-g$	additive inverse of g, 51
$\lvert G \rvert$	order of the group G, 60
$\lvert g \rvert$	order of the element g, 60
$H \leq G$	subgroup inclusion, 61
$H < G$	subgroup $H \neq G$, 61
$\langle a \rangle$	$\{a^n \mid n \in Z\}$, cyclic group generated by a, 65
$Z(G)$	$\{a \in G \mid ax = xa$ for all x in $G\}$, the center of G, 66

$C(a)$	$\{g \in G \mid ga = ag\}$, the centralizer of a in G, 68
$\langle S \rangle$	subgroup generated by the set S, 71
$C(H)$	$\{x \in G \mid xh = hx$ for all $h \in H\}$, the centralizer of H, 71
$\phi(n)$	Euler phi function of n, 83
S_n	group of one-to-one functions from $\{1, 2, \ldots, n\}$ to itself, 95
A_n	alternating group of degree n, 95
$G \approx \overline{G}$	G and $\overline{G}$ are isomorphic, 121
ϕ_a	mapping given by $\phi_a(x) = axa^{-1}$ for all x, 128
$\mathrm{Aut}(G)$	group of automorphisms of the group G, 129
$\mathrm{Inn}(G)$	group of inner automorphisms of G, 129
aH	$\{ah \mid h \in H\}$, 138
aHa^{-1}	$\{aha^{-1} \mid h \in H\}$, 138
$\lvert G{:}H \rvert$	the index of H in G, 142
HK	$\{hk \mid h \in H, k \in K\}$, 144
$\mathrm{stab}_G(i)$	$\{\phi \in G \mid \phi(i) = i\}$, the stabilizer of i under the permutation group G, 146
$\mathrm{orb}_G(i)$	$\{\phi(i) \mid \phi \in G\}$, the orbit of i under the permutation group G, 146
$G_1 \oplus G_2 \oplus \cdots \oplus G_n$	external direct product of groups $G_1, G_2, \ldots, G_n$, 156
$U_k(n)$	$\{x \in U(n) \mid x \bmod k = 1\}$, 160
$H \triangleleft G$	H is a normal subgroup of G, 174
G/H	factor group, 176
$H \times K$	internal direct product of H and K, 183
$H_1 \times H_2 \times \cdots \times H_n$	internal direct product of $H_1, \ldots, H_n$, 184
$\mathrm{Ker}\, \phi$	kernel of the homomorphism ϕ, 194
$\phi^{-1}(g')$	inverse image of g' under ϕ, 196
$\phi^{-1}(\overline{K})$	inverse image of $\overline{K}$ under ϕ, 197
$Z[x]$	ring of polynomials with integer coefficients, 228
$M_2(Z)$	ring of all 2×2 matrices with integer entries, 228
$R_1 \oplus R_2 \oplus \cdots \oplus R_n$	direct sum of rings, 229
nZ	ring of multiples of n, 231
$Z[i]$	ring of Gaussian integers, 231
$U(R)$	group of units of the ring R, 233
$\mathrm{char}\, R$	characteristic of R, 240
$\langle a \rangle$	principal ideal generated by a, 250
$\langle a_1, a_2, \ldots, a_n \rangle$	ideal generated by $a_1, a_2, \ldots, a_n$, 250
R/A	factor ring, 250
$A + B$	sum of ideals A and B, 256
AB	product of ideals A and B, 257
$\mathrm{Ann}(A)$	annihilator of A, 258
$N(A)$	nil radical of A, 258
$F(x)$	field of quotients of $F[x]$, 269
$R[x]$	ring of polynomials over R, 276
$\deg f(x)$	degree of the polynomial, 278
$\Phi_p(x)$	pth cyclotomic polynomial, 294
$M_2(Q)$	ring of 2×2 matrices over Q, 330
$\langle v_1, v_2, \ldots, v_n \rangle$	subspace spanned by $v_1, v_2, \ldots, v_n$, 331
$F(a_1, a_2, \ldots, a_n)$	extension of F by $a_1, a_2, \ldots, a_n$, 341

$f'(x)$	the derivative of $f(x)$, 346
$[E:F]$	degree of E over F, 356
$\mathrm{GF}(p^n)$	Galois field of order p^n, 368
$\mathrm{GF}(p^n)^*$	nonzero elements of $\mathrm{GF}(p^n)$, 369
$\mathrm{cl}(a)$	$\{xax^{-1} \mid x \in G\}$, the conjugacy class of a, 387
n_p	the number of Sylow p-subgroups of a group, 393
$W(S)$	set of all words from S, 424
$\langle a_1, a_2, \ldots, a_n \mid w_1 = w_2 = \cdots = w_t \rangle$	group with generators $a_1, a_2, \ldots, a_n$ and relations $w_1 = w_2 = \cdots = w_t$, 426
Q_4	quarternions, 430
Q_6	dicyclic group of order 12, 430
D_∞	infinite dihedral group, 431
$\mathrm{fix}(\phi)$	$\{i \in S \mid \phi(i) = i\}$, elements fixed by ϕ, 474
$\mathrm{Cay}(S:G)$	Cayley digraph of the group G with generating set S, 482
$k * (a, b, \ldots, c)$	concatenation of k copies of $(a, b, \ldots, c)$, 490
(n, k)	linear code, k-dimensional subspace of F^n, 508
F^n	$F \oplus F \oplus \cdots \oplus F$, direct product of n copies of the field F, 508
$d(u, v)$	Hamming distance between vectors u and v, 509
$\mathrm{wt}(u)$	the number of nonzero components of the vector u (the Hamming weight of u), 509
$\mathrm{Gal}(E/F)$	the automorphism group of E fixing F, 531
E_H	fixed field of H, 531
$\Phi_n(x)$	nth cyclotomic polynomial, 548

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In memory of my brother.