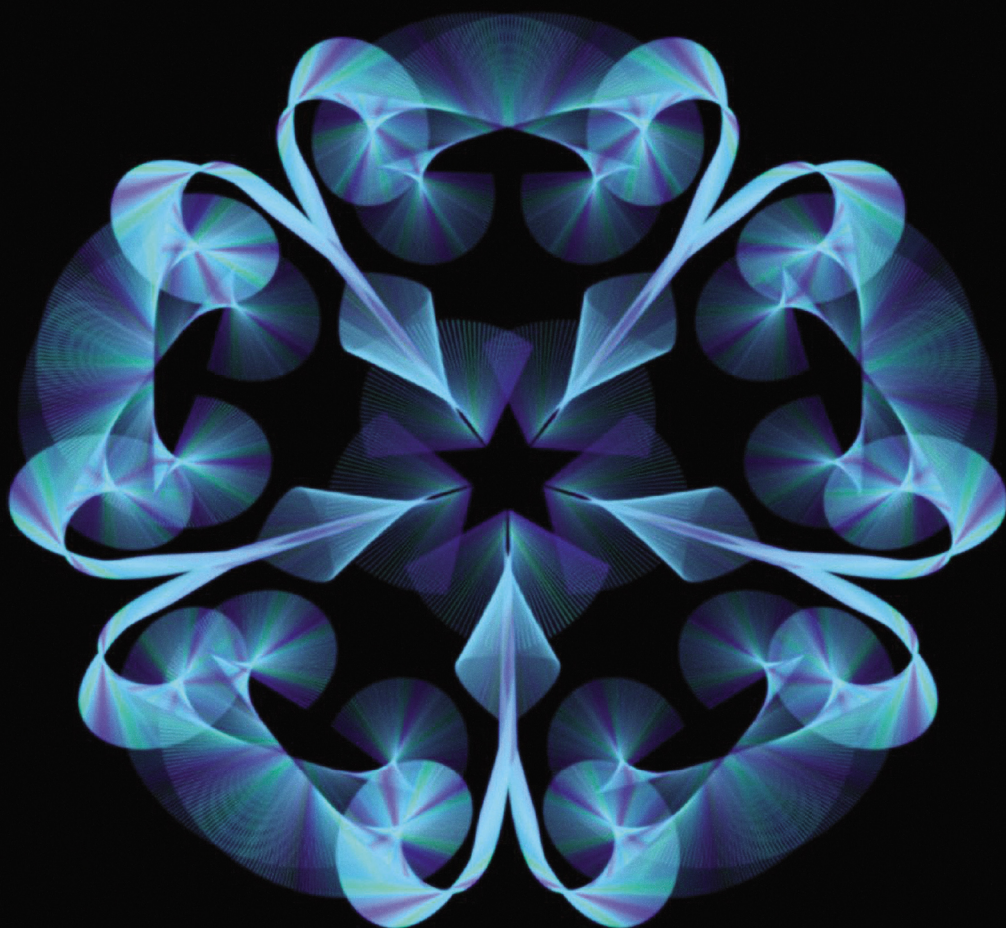


Joseph A. Gallian

CONTEMPORARY ABSTRACT ALGEBRA

Ninth Edition



Notations

(The number after the item indicates the page where the notation is defined.)

SET THEORY

$\cap_{i \in I} S_i$	intersection of sets S_i , $i \in I$
$\cup_{i \in I} S_i$	union of sets S_i , $i \in I$
$[a]$	$\{x \in S \mid x \sim a\}$, equivalence class of S containing a , 18
$ S $	number of elements in the set of S

SPECIAL SETS

$\mathbf{Z}$	integers, additive groups of integers, ring of integers
$\mathbf{Q}$	rational numbers, field of rational numbers
$\mathbf{Q}^+$	multiplicative group of positive rational numbers
F^*	set of nonzero elements of F
$\mathbf{R}$	real numbers, field of real numbers
$\mathbf{R}^+$	multiplicative group of positive real numbers
$\mathbf{C}$	complex numbers

FUNCTIONS AND ARITHMETIC

f^{-1}	inverse of the function f
$t \mid s$	t divides s , 3
$t \nmid s$	t does not divide s , 3
$\gcd(a, b)$	greatest common divisor of the integers a and b , 4
$\text{lcm}(a, b)$	least common multiple of the integers a and b , 6
$ a + b $	$\sqrt{a^2 + b^2}$, 13
$\phi(a)$	image of a under ϕ , 20
$\phi: A \rightarrow B$	mapping of A to B , 21
$gf, \alpha\beta$	composite function, 21

ALGEBRAIC SYSTEMS

D_4	group of symmetries of a square, dihedral group of order 8, 33
D_n	dihedral group of order $2n$, 34
e	identity element, 43
$\mathbf{Z}_n$	group $\{0, 1, \dots, n-1\}$ under addition modulo n , 44
$\det A$	the determinant of A , 45
$U(n)$	group of units modulo n (that is, the set of integers less than n and relatively prime to n under multiplication modulo n), 46
$\mathbf{R}^n$	$\{(a_1, a_2, \dots, a_n) \mid a_1, a_2, \dots, a_n \in \mathbf{R}\}$, 47
$SL(2, F)$	group of 2×2 matrices over F with determinant 1, 47
$GL(2, F)$	2×2 matrices of nonzero determinants with coefficients from the field F (the general linear group), 48
g^{-1}	multiplicative inverse of g , 51
$-g$	additive inverse of g , 51
$ G $	order of the group G , 60
$ g $	order of the element g , 60
$H \leq G$	subgroup inclusion, 61
$H < G$	subgroup $H \neq G$, 61
$\langle a \rangle$	$\{a^n \mid n \in \mathbf{Z}\}$, cyclic group generated by a , 65
$Z(G)$	$\{a \in G \mid ax = xa \text{ for all } x \text{ in } G\}$, the center of G , 66

$C(a)$	$\{g \in G \mid ga = ag\}$, the centralizer of a in G , 68
$\langle S \rangle$	subgroup generated by the set S , 71
$C(H)$	$\{x \in G \mid xh = hx \text{ for all } h \in H\}$, the centralizer of H , 71
$\phi(n)$	Euler phi function of n , 83
S_n	group of one-to-one functions from $\{1, 2, \dots, n\}$ to itself, 95
A_n	alternating group of degree n , 95
$G \approx \bar{G}$	G and $\bar{G}$ are isomorphic, 121
ϕ_a	mapping given by $\phi_a(x) = axa^{-1}$ for all x , 128
$\text{Aut}(G)$	group of automorphisms of the group G , 129
$\text{Inn}(G)$	group of inner automorphisms of G , 129
aH	$\{ah \mid h \in H\}$, 138
aHa^{-1}	$\{aha^{-1} \mid h \in H\}$, 138
$ G:H $	the index of H in G , 142
HK	$\{hk \mid h \in H, k \in K\}$, 144
$\text{stab}_G(i)$	$\{\phi \in G \mid \phi(i) = i\}$, the stabilizer of i under the permutation group G , 146
$\text{orb}_G(i)$	$\{\phi(i) \mid \phi \in G\}$, the orbit of i under the permutation group G , 146
$G_1 \oplus G_2 \oplus \dots \oplus G_n$	external direct product of groups $G_1, G_2, \dots, G_n$, 156
$U_k(n)$	$\{x \in U(n) \mid x \bmod k = 1\}$, 160
$H \triangleleft G$	H is a normal subgroup of G , 174
G/H	factor group, 176
$H \times K$	internal direct product of H and K , 183
$H_1 \times H_2 \times \dots \times H_n$	internal direct product of $H_1, \dots, H_n$, 184
$\text{Ker } \phi$	kernel of the homomorphism ϕ , 194
$\phi^{-1}(g')$	inverse image of g' under ϕ , 196
$\phi^{-1}(\bar{K})$	inverse image of $\bar{K}$ under ϕ , 197
$\mathbb{Z}[x]$	ring of polynomials with integer coefficients, 228
$M_2(\mathbb{Z})$	ring of all 2×2 matrices with integer entries, 228
$R_1 \oplus R_2 \oplus \dots \oplus R_n$	direct sum of rings, 229
$n\mathbb{Z}$	ring of multiples of n , 231
$\mathbb{Z}[i]$	ring of Gaussian integers, 231
$U(R)$	group of units of the ring R , 233
$\text{char } R$	characteristic of R , 240
$\langle a \rangle$	principal ideal generated by a , 250
$\langle a_1, a_2, \dots, a_n \rangle$	ideal generated by $a_1, a_2, \dots, a_n$, 250
R/A	factor ring, 250
$A + B$	sum of ideals A and B , 256
AB	product of ideals A and B , 257
$\text{Ann}(A)$	annihilator of A , 258
$N(A)$	nil radical of A , 258
$F(x)$	field of quotients of $F[x]$, 269
$R[x]$	ring of polynomials over R , 276
$\deg f(x)$	degree of the polynomial, 278
$\Phi_p(x)$	p th cyclotomic polynomial, 294
$M_2(Q)$	ring of 2×2 matrices over Q , 330
$\langle v_1, v_2, \dots, v_n \rangle$	subspace spanned by $v_1, v_2, \dots, v_n$, 331
$F(a_1, a_2, \dots, a_n)$	extension of F by $a_1, a_2, \dots, a_n$, 341

$f'(x)$	the derivative of $f(x)$, 346
$[E:F]$	degree of E over F , 356
$\text{GF}(p^n)$	Galois field of order p^n , 368
$\text{GF}(p^n)^*$	nonzero elements of $\text{GF}(p^n)$, 369
$\text{cl}(a)$	$\{xax^{-1} \mid x \in G\}$, the conjugacy class of a , 387
n_p	the number of Sylow p -subgroups of a group, 393
$W(S)$	set of all words from S , 424
$\langle a_1, a_2, \dots, a_n \mid w_1 = w_2 = \dots = w_t \rangle$	group with generators $a_1, a_2, \dots, a_n$ and relations $w_1 = w_2 = \dots = w_t$, 426
Q_4	quaternions, 430
Q_6	dicyclic group of order 12, 430
D_∞	infinite dihedral group, 431
$\text{fix}(\phi)$	$\{i \in S \mid \phi(i) = i\}$, elements fixed by ϕ , 474
$\text{Cay}(S;G)$	Cayley digraph of the group G with generating set S , 482
$k * (a, b, \dots, c)$	concatenation of k copies of $(a, b, \dots, c)$, 490
(n, k)	linear code, k -dimensional subspace of F^n , 508
F^n	$F \oplus F \oplus \dots \oplus F$, direct product of n copies of the field F , 508
$d(u, v)$	Hamming distance between vectors u and v , 509
$\text{wt}(u)$	the number of nonzero components of the vector u (the Hamming weight of u), 509
$\text{Gal}(E/F)$	the automorphism group of E fixing F , 531
E_H	fixed field of H , 531
$\Phi_n(x)$	n th cyclotomic polynomial, 548

Contemporary Abstract Algebra

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NINTH EDITION

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In memory of my brother.