

# TRANSFORMERS AND INDUCTORS

FOR POWER ELECTRONICS

THEORY, DESIGN AND APPLICATIONS



W. G. HURLEY AND W. H. WÖLFLE

 WILEY

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*To Our Families*

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