



Hybrid Phonons in
Nanostructures

B. K. RIDLEY

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First Edition

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Preface

Crystalline nanostructures confine electrons and quantize their energies, which effect has led to the nomenclature quantum wells, quantum wires, and quantum dots. They also confine lattice waves. In the case of acoustic modes propagating in wave guides, this confinement is well understood by classical physics. This is not the case for optical modes of vibration. In acoustic wave guides and, indeed, in nanostructures, the normal modes of vibration are determined by the satisfaction of the classical connection rules—continuation of particle displacement and continuity of stress—at the boundaries. In order to satisfy these rules, a mode may have to combine with a mode of different polarization to form a hybrid. In many cases, a longitudinally polarized acoustic (LA) mode must liase with a transversely polarized acoustic (TA) mode to form a viable normal mode of the system. Such hybrid modes are commonplace in confining structures and have been known from the end of the nineteenth century. This is not the case for optical modes. Relative to acoustic modes, optical modes are but lately come, and their properties not as well defined or understood. This lacuna has been one of the motives for writing this book.

An understanding of optical modes in room-temperature devices is of some importance since they are, in polar structures, the principal source of electrical resistance. Acoustic modes are also important in that respect, but the shared frequencies between barrier and well make the confinement of acoustic modes at once more intricate and more open to simplification. On the one hand, reflection and transmission at the boundary lead to a rich family of mode patterns that include guided modes and interface waves. On the other hand, as regards the electron–phonon interaction, it may be sufficient for many purposes to disregard the intricacies entirely and treat the entire acoustic spectrum as bulk-like. This is not an easily justifiable option for optical modes, given the disparity of frequency between barrier and well that is commonly encountered. In that respect, optical modes present a problem. What exactly are the mechanical connection rules? In the case of polar modes, there are the usual electromagnetic boundary conditions as well as those mysterious rules associated with the elasticity of the lattice. Can the electromagnetic boundary conditions be sufficient? In other words, can the crystal be regarded simply as a dielectric continuum? For those interested solely in estimating the strength of the interaction between electrons and polar optical modes, the dielectric continuum (DC) model provides a simpler alternative to hybrid theory, an alternative that is not without some theoretical justification. Nevertheless, a crystal is not a simple dielectric continuum. If the physics of nanostructures is to see the semiconductor as a continuum, it must be a continuum that possesses both elastic and dielectric properties, inhabited by hybrid lattice vibrations, both

acoustic and optical, along with confined electrons. These constitute the essential elements of the nanostructures and their interaction that will be described here.

Inevitably, such a description generates many equations, which many students of nanostructure physics may find somewhat indigestible. As one who prefers intuition to rigour (for better or worse), and who observes somewhat distantly the purely formal mathematical approach with some admiration, I have much sympathy with this attitude, but the student should know that the equations would be much more indigestible were they to portray a truly rigorous reality that took into account the natural anisotropy of semiconductor crystals. For simplicity, the hybrid modes that are described here are creatures of purely isotropic solids, in which modes are polarized purely longitudinally or purely transversely. Moreover, they are all long-wavelength modes, which allow a clear distinction to be made between optical and acoustic. Such approximations are acceptable for the Groups IV and III-V cubic semiconductors, but not for the hexagonal II-VI materials, which are highly anisotropic and, moreover, exhibit more than one optical mode. The book has been written with cubic semiconductors very much in mind.

Some parts of this book were written during and after moving house from Essex to Herefordshire (often to the despair of my wife). I suspect it has kept me sane during what most think of as one of the most traumatic events of life. Perhaps physics is to be recommended as a balm in troublesome times. My wife, bless her, doubts it.

Pembridge 2016

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