VIETNAM ACADEMY OF SCIENCE AND TECHNOLOGY
INSTITUTE OF MATHEMATICS

NGUYEN THANH QUI

Coderivatives of Normal Cone Mappings and Applications

DOCTORAL DISSERTATION IN MATHEMATICS

HANOI - 2014

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Supervisors:
1. Prof. Dr. Hab. Nguyen Dong Yen
2. Dr. Bui Trong Kien

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To my beloved parents and family members

Confirmation

This dissertation was written on the basis of my research works carried at Institute of Mathematics (VAST, Hanoi) under the supervision of Professor Nguyen Dong Yen and Dr. Bui Trong Kien. All the results presented have never been published by others.

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The author

Nguyen Thanh Qui

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Contents

Table of Notations

$I\!N := \{1, 2, \ldots\}$	set of positive natural numbers
$\emptyset$	empty set
$I\!R$	set of real numbers
$I\!R_{++}$	set of $x \in I\!R$ with $x > 0$
$I\!R_{+}$	set of $x \in I\!R$ with $x \geq 0$
$I\!R_{-}$	set of $x \in I\!R$ with $x \leq 0$
$\overline{I\!R} := I\!R \cup \{\pm\infty\}$	set of generalized real numbers
$\|x\|$	absolute value of $x \in I\!R$
$I\!R^n$	n-dimensional Euclidean vector space
$\\|x\\|$	norm of a vector x
$I\!R^{m\times n}$	set of $m \times n$-real matrices
$\det A$	determinant of a matrix A
$A^\top$	transposition of a matrix A
$\\|A\\|$	norm of a matrix A
X^*	topological dual of a norm space X
$\langle x^*, x\rangle$	canonical pairing
$\langle x, y\rangle$	canonical inner product
$\widehat{(u, v)}$	angle between two vectors u and v
$B(x, \delta)$	open ball with centered at x and radius δ
$\bar{B}(x, \delta)$	closed ball with centered at x and radius δ
B_X	open unit ball in a norm space X
$\bar{B}_X$	closed unit ball in a norm space X
$\mathrm{pos}\Omega$	convex cone generated by Ω
$\mathrm{span}\Omega$	linear subspace generated by Ω
$\mathrm{dist}(x; \Omega)$	distance from x to Ω
$\{x_k\}$	sequence of vectors
$x_k \to x$	x_k converges to x in norm topology
$x_k^* \xrightarrow{w^*} x^*$	x_k^* converges to x^* in weak* topology

$\forall x$	for all x
$x := y$	x is defined by y
$\widehat{N}(x;\Omega)$	Fréchet normal cone to Ω at x
$N(x;\Omega)$	limiting normal cone to Ω at x
$f : X \to Y$	function from X to Y
$f'(x)$, $\nabla f(x)$	Fréchet derivative of f at x
$\varphi : X \to \overline{I\!R}$	extended-real-valued function
$\mathrm{dom}\varphi$	effective domain of φ
$\mathrm{epi}\varphi$	epigraph of φ
$\partial\varphi(x)$	limiting subdifferential of φ at x
$\partial^2\varphi(x,y)$	limiting second-order subdifferential of φ at x relative to y
$F : X \rightrightarrows Y$	multifunction from X to Y
$\mathrm{dom}F$	domain of F
$\mathrm{rge}F$	range of F
$\mathrm{gph}F$	graph of F
$\mathrm{ker}F$	kernel of F
$\widehat{D}^*F(x,y)$	Fréchet coderivative of F at (x,y)
$D^*F(x,y)$	Mordukhovich coderivative of F at (x,y)