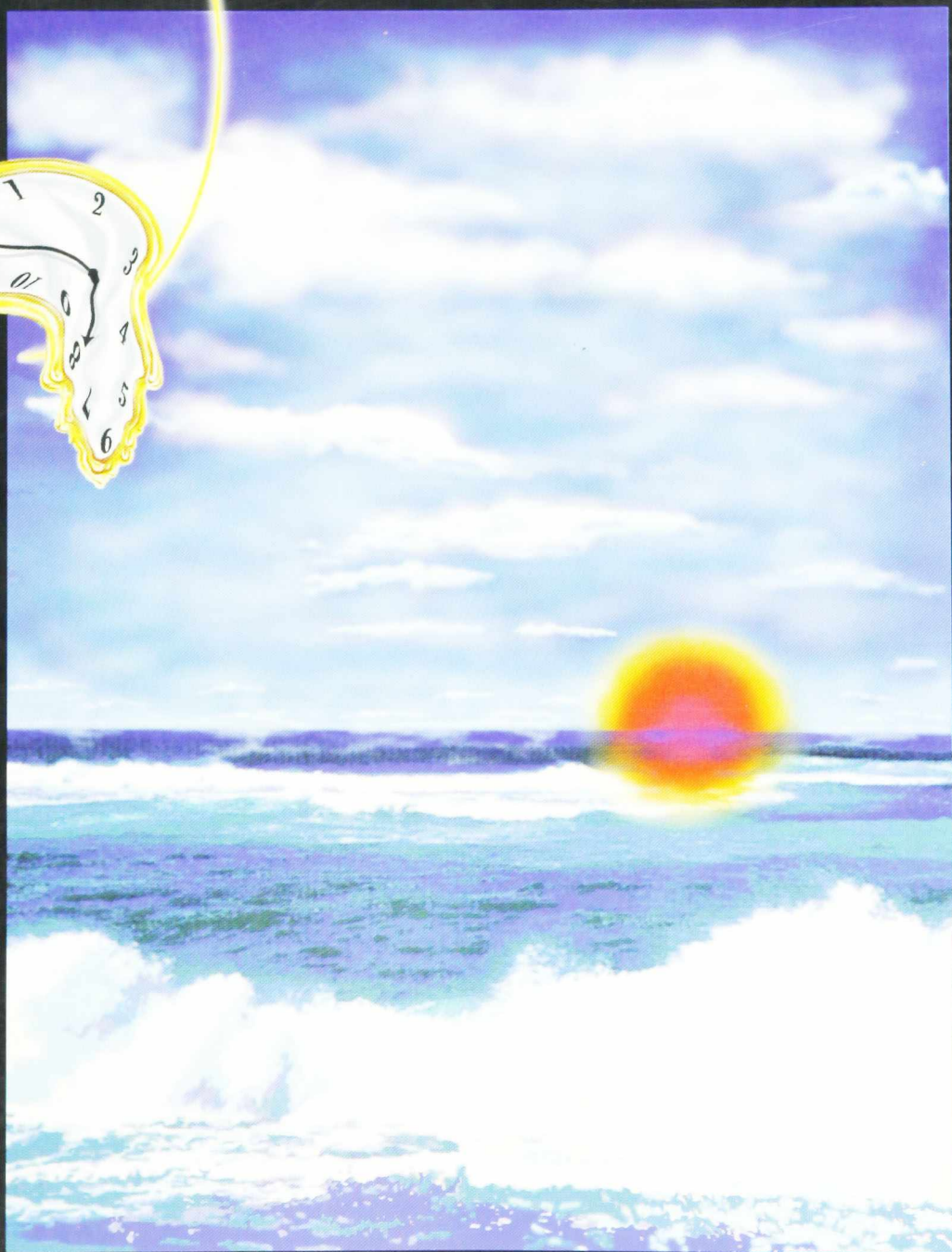




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# INSTANTONS IN GAUGE THEORIES



Edited by

**M. Shifman**

**World Scientific**

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*Theoretical Physics Institute  
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## PREFACE

In this volume you will find a compilation of works which, taken together, give a complete and consistent presentation of instanton calculus in non-Abelian gauge theories, as it exists now. Some of the papers reproduced are instanton classics. Among other things, they show from a historical perspective how the instanton solution has been found, the motivation behind it and how the physical meaning of instantons has been revealed. Other papers are devoted to different aspects of the instanton formalism. Only topics which have a direct relation to the formalism *per se* are included. No attempt is made to discuss numerous applications in non-Abelian gauge theories or other models. The papers are organized into several sections that are linked both logically and historically, accompanied by brief comments.

*M. Shifman*  
*November 28, 1993*

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