A NEW STUDY ON OPTIMUM CALCULATION OF PARTIAL TRANSMISSION RATIOS OF COUPLED PLANETARY GEAR SETS

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ABSTRACT

This article presents a new study on optimum calculation of the partial ratios of coupled planetary gear sets for getting minimum radial size of the gear sets. In this article, based on moment equilibrium condition of a mechanic system including two-row planetary gear sets and their regular resistance conditions, an efficient model for calculating the partial ratios of coupled planetary gear sets was proposed. In addition, by giving this explicit model, the partial ratios can be calculated accurately and simply.

Keywords: *Transmission ratio, Gearbox design, Optimum design, Planetary gearbox.*

INTRODUCTION

In gearbox design as well as in planetary gearbox design, one of the most important problems is optimum determination of partial transmission ratios of a gearbox. This is because the partial ratios are main factors which affect the size, the dimension, the mass, and the cost of the gearbox. Therefore, optimum calculation of the partial ratios of gearboxes has been subjected to many studies.

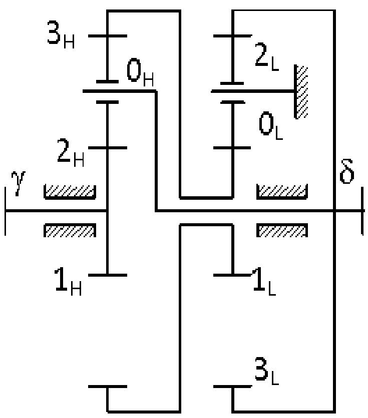

Figure. 1. *Schema of a couled planetary gear set*

Until now, many researches have been done on the calculation of the partial ratios of gearboxes. This type of tasks has been solved with different gearboxes such as helical gearboxes (in [1], [2], [3], [4] and [5]), bevel and bevel – helical gearboxes (in [1], [3], [5] and [6]) and worm and worm-helical gearboxes (in [5], [7] and [8]).

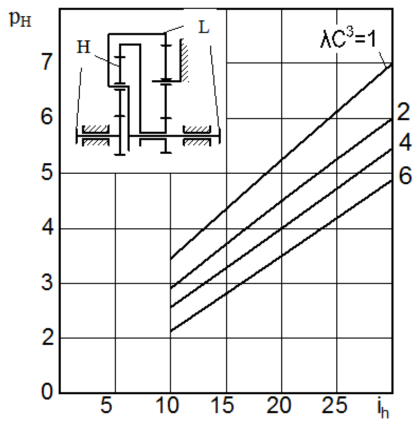

Figure. 2. *Graph for finding partial ratios*

Also, the optimum partial ratios of gearboxes can be found by different ways: By graph method (in [1], [3], [8]), by "practical method" (the ratios were given based on analyzing practical data (in [5])) and by models (based theoritical (in [2] and [6]) or regression models (in [4] and [6]).

From above analysis, it is clear that, there are many studies have been conducted for calculating the optimum partial ratios of gearboxes. However, there was only a study [1] on this problem for planetary gearboxes. In the study, the partial ratios of planetary of three schemas of planetary gearboxes were

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predicted by graphs. For example, for coupled planetary gear sets in the Figure 1, the partion ratio of the low-speed row gear unit p_L (p_L = $d3_L/d1_L$) can be predicted by the graph in the Figure 2. After that, the partion ratio of the high-speed row gear unit p_H (p_H=$d3_HL/d_H$) can be calculated based on the values of u_h and p_L ($dw1_L$, $dw1_H$, $dw3_L$ and $dw3_H$ are pitch diameters (mm)).

This article introduces a new study on optimum calculation of partial transmission ratios of coupled planetary gear sets for getting the minimum radial size of the gear sets.

OPTIMUM DETERMINATION OF PARTIAL TRANSMISSION RATIOS OF COUPLED PLANETARY GEARBOXES

For the low-speed row of the coupled planetary gearbox (see Figure 1), the design equation for the pitting resistance can be given by the following equation [8]:

$$\sigma_{HL} = Z_{ML} \cdot Z_{HL} \cdot Z_{\varepsilon L} \cdot \sqrt{\frac{2T_{1L} \cdot K_{HL} \cdot \sqrt{u_L + 1}}{b_{wL} \cdot d_{w1L}^2 \cdot q \cdot u_L}} \le [\sigma_{HL}] \quad (1)$$

Where, Z_{ML}, Z_{HL} and $Z_{\varepsilon L}$ are coefficients which consider the effects of the gear material, contact surface shape, and contact ratio of the first gear unit;T_{1L} is the torque of the driving shaft (Nm), $[\sigma_{HL}]$ is allowable contact stresses of the low-speed row of the planetary gearbox.

From (1) the allowable torque (Nm) of the driving shaft of the low-speed unit can be found:

$$[T_{1L}] = \frac{b_{wL} \cdot d_{w1L}^2 \cdot q_L \cdot u_L \cdot [\sigma_{HL}]^2}{2 \cdot (u_L + 1) \cdot K_{HL} \cdot (Z_{ML} \cdot Z_{HL} \cdot Z_{\varepsilon L})^2} \quad (2)$$

From gear theory, we have:

$$b_{wL} = \psi_{baL} \cdot a_{wL} \quad (3)$$

$$a_{\mathrm{wL}} = \frac{(u_L + 1) \cdot d_{\mathrm{w1L}}}{2} \quad (4)$$

Where, b_{wL} and a_{wL} are the face with (m) and the center distance (m) of the low-speed unit.

Substituting (3) and (4) into (2) we get:

$$[T_{1L}] = \frac{\psi_{baL} \cdot d_{w1L}^3 \cdot q_L \cdot u_L \cdot [K_{0L}]}{4} \quad (5)$$

In which:

$$[K_{0L}] = \frac{[\sigma_{HL}]^2}{K_{HL}(Z_{ML}.Z_{HL}.Z_{\varepsilon L})}$$

Put $u_L = \frac{Z_{2L}}{Z_{1L}} = \frac{(p_L - 1)}{2}$ into (5) we get:

$$[T_{1L}] = \frac{\psi_{baL} \cdot d_{w1L}^3 \cdot q_L \cdot (p_L - 1) \cdot [K_{0L}]}{8} \quad (6)$$

Calculating in the same way, the allowable torque of the driving shaft of the high-speed row of the planetary gearbox was found:

$$[T_{1H}] = \frac{\psi_{baH} \cdot d_{w1H}^3 \cdot q_H \cdot (p_H - 1) \cdot [K_{0H}]}{8} \quad (7)$$

From (6) and (7), the rate of $[T_{1H}]/[T_{1L}]$ is:

$$\frac{[T_{1H}]}{[T_{1L}]} = \frac{\psi_{baH}}{\psi_{baL}} \cdot \frac{[K_{0H}]}{[K_{0L}]} \cdot \frac{q_H}{q_L} \cdot \left(\frac{d_{\mathrm{w1H}}}{d_{\mathrm{w1L}}}\right)^3 \cdot \left(\frac{p_H - 1}{p_L - 1}\right) \quad (8)$$

Substituting $d_{1L} = d_{w3L} / p_L$ and $d_{1H} = d_{w3H} / p_H$ into (8), the equation becomes:

$$\frac{[T_{1H}]}{[T_{1L}]} = \frac{\psi_{baH}}{\psi_{baL}} \cdot \frac{[K_{0H}]}{[K_{0L}]} \cdot \frac{q_H}{q_L} \cdot \left(\frac{d_{\mathrm{w3H}}}{d_{\mathrm{w3L}}}\right)^3 \cdot \left(\frac{p_L}{p_H}\right)^3 \cdot \left(\frac{p_H - 1}{p_L - 1}\right) \quad (9)$$

From above equation we have:

$$\frac{[T_{1H}]}{[T_{1L}]} = \frac{c_k \cdot c^3}{c_x} \cdot \frac{q_H}{q_L} \cdot \left(\frac{p_L}{p_H}\right)^3 \cdot \left(\frac{p_H - 1}{p_L - 1}\right) \quad (10)$$

In the above equations, $c_k = [K_{0H}]/[K_{0L}]$, $c = d_{w3H} / d_{w3L}$, $c_x = \psi_{baL} / \psi_{baH}$; d_{w3H} and d_{w3L} are pitch diameters of ring gears of high and low-speed units.

With the coupled planetary gearbox in Figure 1 we have [1]:

$$[T_{1H}] = -T_{3L} \cdot \frac{1}{p_H \cdot (p_L + 1) + 1} \quad (11)$$

$$[T_{3H}] = [T_{1L}] = -T_{3L} \cdot \frac{p_H}{p_H \cdot (p_L + 1) + 1} \quad (12)$$

From (11) and (12) we get:

$$[T_{1H}]/[T_{1L}] = \frac{1}{p_H} \quad (13)$$

Also, uh can be calculated by the following equation [1]:

$$u_h = u_{\gamma\delta} = 1 + p_H\left(p_L + 1\right) \quad (14)$$

From (14) the the ratio p_L can be given as:

$$p_H = \frac{u_h - 1}{p_L + 1} \quad (15)$$

Substituting (13) and (15) into (10) we have:

$$\frac{c_k \cdot c^3}{c_x} \cdot \frac{q_H}{q_L} \cdot \frac{p_L^3(p_L+1)(u_h - p_L - 2)}{(u_h - 1)^2(p_L - 1)} = 1 \quad (16)$$

In practice, $c = 1\ldots1.2$ [1]. In order to get the minimum radial size of the gearbox $c = 1$ [1]. In addition, we can choose $c_k \approx 1$ and $c_x = \psi_{baL}/\psi_{baH} = 1\ldots1.3$ [8]. Also, the number of planetary gears of each unit is generally chosen as $3\left(q_L = q_H = 3\right)$. Therefore, equation (16) can be rewritten as follows:

$$\frac{p_L^3(p_L+1)(u_h - p_L - 2)}{c_x(u_h - 1)^2(p_L - 1)} = 1 \quad (17)$$

To find the value of p_L which depends on the total ratio of the gearbox u_h and the coefficient c_x a computer program was built. The data used in the program as follows: $u_h = 15\ldots60$ and $c_x = \psi_{ba_L}/\psi_{ba_H} = 1\ldots1.3$. From the results of the program, the following regression model (with the coefficient of determination was $R^2 = 0.91$) was found for the optimal values of p_L:

$$p_L = 0.4967 \cdot \left(\psi_{ba_L}/\psi_{ba_H}\right)^{0.5141} \cdot u_h^{0.4562} \quad (18)$$

Equation (18) is used to determine the partial ratio of the low-speed row gear unit of the gearbox p_L. After finding p_L, the partion ratio of the high-speed row gear unit p_H ($p_H = d_{w3_H}/d_{1_H}$) can be determined based on the values of u_h and p_L by equation (18) .

CONCLUSIONS

The minimum radial size of the coupled planetary gearbox can be obtained based on theoretical analysis and regression method. Model for calculation of the optimum partial ratios of doubled planetary gear sets for getting the minimum radial size of the gearboxes have been proposed.

The partial ratios of the gearboxes can be determined accurately and simply by explicit models.

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TÓM TẮT

NGHIÊN CỨU MỚI VỀ TỐI ƯU HÓA TỶ SỐ TRUYỀN CÁC BỘ TRUYỀN TRONG HỘP HÀNH TINH

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Bài báo này giới thiệu một nghiên cứu mới về phân phối tối ưu tỉ số truyền của các cặp bánh răng trong truyền động bánh răng hành tinh 2 cấp. Trong bài báo này, dựa trên điều kiện cân bằng mô men của cơ hệ gồm 2 cấp bánh răng hành tinh và điều kiện sức bền đều của các cấp, các tác giả đã đề xuất các công thức tính toán tỉ số truyền tối ưu cho từng cấp của hệ. Bằng việc đưa ra các công thức dưới dạng hàm hiển, tỉ số truyền tối ưu của các cấp có thể xác định một cách nhanh chóng và đơn giản.

Từ khóa: *Tỉ lệ truyền, thiết kế hộp tốc độ, thiết kế tốt ưu, hộp hành tinh.*

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