KINEMATIC AND DYNAMIC MODELING AND SIMULATION OF CRANK MECHANISM OF AUTOMOBILE ENGINE

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SUMMARY

The study aims to conduct the kinematic and dynamic simulation of the four stroke engine in MATLAB software. A method of modeling which allows calculating the dynamics and kinematics of crank mechanism of automobile engine has been developed. Typical dynamic and kinematic performance characteristics of 4-cylinders engine are shown.

Key words: *kinematics, dynamics, crank mechanism, simulation*

INTRODUCTION

The principle behind an internal combustion engine (ICE) is that air and fuel are mixed and burnt inside the cylinder to generate work. The combustion pushes the piston, which transfer the translational movement through the connecting rod, to a rotational movement on the crank shaft. During each cycle, the piston moves continuously back and forth between its top and bottom positions, commonly known as the Top Dead Center (TDC) and the Bottom Dead Center (BDC), respectively. Kinematic analysis is important to understand the position, velocity and acceleration of each linkage during the working of mechanism. The essentiality of dynamic analysis is to understand dynamic behavior of each link, during the working of mechanism.

Analysis of the publications [7-15] showed that most of works focus on computational simulation and thermodynamic cycle [11, 12] or calculate the parameters of the engine with valve timing, valve-contour or contour gas distribution [13, 14]. Some other works using specialized software such as Adams, Saber, Scilab, GT-power... to simulate the entire engine [9,10] or crank mechanism [7,8]. Nonetheless, there is no a complete study of the kinetics and dynamics of the 4-stroke engine with different numbers of cylinder. In the following discussion, we refer to the kinematics and dynamics of the crank mechanism, performed with the aid of MATLAB software.

KINEMATICS AND DYNAMICS OF CRANK MECHANISM

Figure 1 shows a diagram of a *central* crank mechanism in which the axis of the cylinder intersects that of the crankshaft [1,2]. The following notation is used in this figure: φ – *angle of crank travel counted from the cylinder axis in the direction of clockwise crankshaft rotation, when* $\varphi = 0°$ *the piston is at TDC (point* A'*),* $\varphi = 180°$ *the piston is at BDC (point* A''*);* β *– angle between the connecting rod and the cylinder axis;* ω *– angular velocity of crankshaft rotation;* $S = 2R$ *– piston stroke;* R *– crank radius;* l *– connecting rod length;* $\lambda = R/l$ *–ratio between crank radius and connecting rod length.*

The expression for the piston travel s from its initial position at TDC when the crank turns through the angle φ, According to Fig. 1 can be expressed as:

$$s = (R+l) - (R\cos\varphi + l\cos\beta) = \left[\left(1+\frac{1}{\lambda}\right) - \left(\cos\alpha + \frac{1}{\lambda}\cos\beta\right)\right]R$$

Determined s with an accuracy up to and including small quantities of the second order, has the following form [1-5]:

$$s = \left[\left(1-\cos\varphi\right) + \frac{\lambda}{4}\left(1-\cos 2\varphi\right)\right]R \qquad (1)$$

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The piston velocity can be determined by taking the derivative of equation (1) with respect to time

$$v_p = \frac{ds}{dt} = \frac{ds}{d\varphi}\frac{d\varphi}{dt} = R\omega\left(\sin\varphi + \frac{\lambda}{2}\sin 2\varphi\right) \quad (2)$$

The *piston acceleration* can be obtained by taking the derivative of expression (2) with respect to time

$$a = \frac{dv_p}{dt} = \frac{dv_p}{d\varphi}\frac{d\varphi}{dt} = R\omega^2\left(\cos\varphi + \lambda\cos 2\varphi\right) \quad (3)$$

During the operation of an engine the crank mechanism parts are acted upon by gas pressure in the cylinder, inertia forces of the reciprocating masses, centrifugal forces, crankcase pressure exerted on the piston, and gravity.

The pressure of the gases in the engine cylinder is usually preceded by thermal calculations [1,2].

To determine the forces of inertia, it is necessary to know the masses of the crank mechanism elements. To simplify the calculations, the actual crank mechanism is replaced by a dynamical equivalent system of lumped masses. All the moving parts are divided into three groups with respect to the nature of their motion:

(a) *Parts reciprocating along the cylinder axis (the piston group)*. The mass of the piston with the rings and pin is assumed to be lumped on the axis of the piston pin and is designated by m_p.

(b) Rotating parts of the crankshaft. Their masses are replaced by a mass $m_{cr.}$ reduced to the crank radius R . The mass of the crankpin m_{cp} with adjacent parts of the webs (Fig. 2a) is assumed to be lumped along the center of the crankpin axis and, since its center of gravity is at a distance R from the shaft axis, this mass need not be reduced. The mass m_{cw} of the middle portion of the crank web over the contour *abcd* with its center of gravity on the radius ρ is reduced to the radius R

$$m_{cw}\rho\omega^2 = m_{cwR}R\omega^2 \ \text{whence} \ m_{cwR} = m_{cw}\frac{\rho}{R}$$

The reduced mass of crank is: $m_{cr} = m_{cp} + 2m_{cwR} = m_{cp} + 2m_{cw}\dfrac{\rho}{R}$

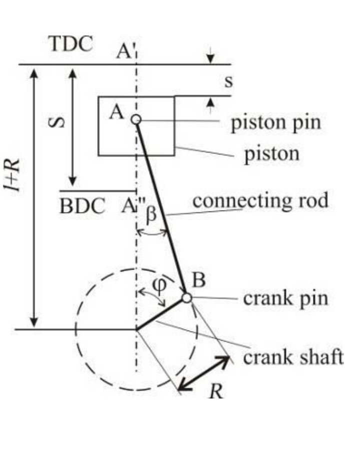

Fig. 1 *Diagrams of central crank mechanism*

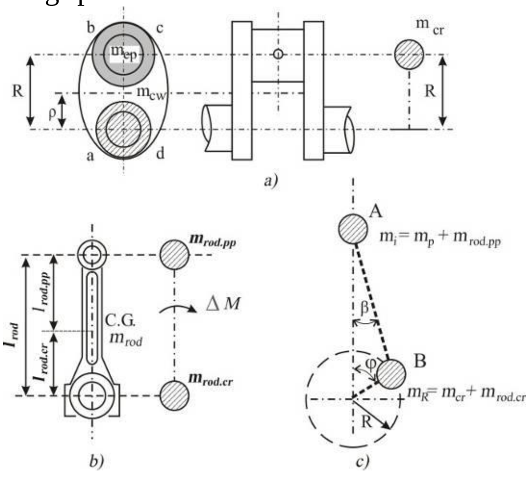

Fig. 2. *Reduction of the crank gear system to a two-mass one:*
a) reduction of crank mass; b)reduction of connecting rod mass; c) reduced system of crank mechanism

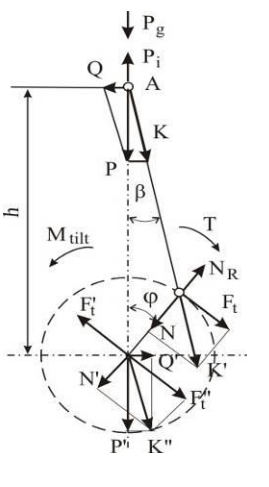

Fig. 3. *Total forces acting in a crank gear*

*(c) Parts performing complex plane-parallel motion (connecting rod group).*The connecting rod is replaced with a certain approximation by a system of two masses statically equivalent to its mass: the mass $m_{rod.pp}$ lumped on the piston pin axis, and the mass $m_{rod\cdot cr}$ on the axis of the crankpin. For this purpose, the mass of the connecting rod m_{rod} is divided into two masses (*Fig.2b*): that referred to the piston pin axis:

$m_{rod.pp} = m_{rod}\dfrac{l_{rod.cr}}{l_{rod}}$ and that refered to the crank axis $m_{rod.cr} = m_{rod}\dfrac{l_{rod.pp}}{l_{rod}}$

To obtain a dynamically equivalent system the following three conditions should be observed: *(i)*. A constant total mass $m_{rod.pp} + m_{rod.cr} = m_{rod}$; (*ii*). A constant position of the center of gravity of the system $m_{rod.pp} l_{rod.pp} - m_{rod.cr} l_{rod.cr} = 0$; *(iii)* A constant moment of inertia of the system with respect to the center of gravity [1-2].

Thus, the entire crank mechanism (Fig. 3*c)* is replaced by a system of two lumped masses connected by rigid weightless links; the reciprocating mass at point *A:* $m_t = m_p + m_{rod.pp}$ and the rotating mass at point *B:* $m_R = m_{cr} + m_{rod.cr}$

In conformity with the adopted system of two masses dynamically equivalent to the crank mechanism, the forces of inertia are reduced to two forces: the force P_t induced by the reciprocating masses and the centrifugal force N_R induced by the rotating ones.

The force of inertia due to reciprocating masses can be represented as the sum of the forces of inertia of the first and second order $P_{i1}; P_{i2}$, which change according to the harmonic law:

$$P_i = -m_i a = -m_i R\omega^2\left(\cos\varphi + \lambda\cos 2\varphi\right) = P_{i1} + P_{i2} \quad (4)$$

where: $P_{i1} = -m_i R\omega^2 \cos\varphi$ and $P_{i2} = -m_i R\omega^2 \lambda \cos 2\varphi$

The centrifugal force of the rotating masses of a crank mechanism

$$N_R = -m_R R\omega^2 \quad (5)$$

N_R is always directed along the crank radius. It is constant in magnitude and applied at the center *B* of the crankpin. The force N_R rotates together with the crank and, not being balanced, is transmitted to the engine supports through the shaft bearing, and the crankcase.

The total force *P* acting on the piston is the initial force

$$P = P_g + P_t \quad (6)$$

The force *P* acting along the cylinder axis (Fig, 3) can be resolved into two components:

$$Q = P\tan\beta \quad (7)$$

$$K = P/\cos\beta \quad (8)$$

The force *K* can be transferred along the line of its action to the center of the crankpin *(K' = K)* and resolved into two components:

$$N = K\cos(\varphi+\beta) = P\frac{\cos(\varphi+\beta)}{\cos\beta} \quad (9)$$

$$F_t = K\sin(\varphi+\beta) = P\frac{\sin(\varphi+\beta)}{\cos\beta} \quad (10)$$

Transfer the normal force *N* along the line of its action to the center of the shaft and denote it as *N*' (i.e., *N*=*N*'*)*. The tangential force F_t will also be transferred to the shaft center *(*$F_t = F'_t = F''_t$*)*. Here a couple of forces (F_t and F'_t*)* appear with the torque *T*:

$$T = F_t R = P\frac{\sin(\varphi+\beta)}{\cos\beta}R = P(\sin\varphi + \tan\beta\cos\varphi)R \quad (11)$$

Since the angle β is small, replace tanβ by $\sin\beta = \lambda\sin\varphi$

$$T \approx P\left(\sin\varphi + \frac{\lambda}{2}\sin 2\varphi\right)R$$

The forces *N*' and F_t'' may be summated. Their resultant *K″* equal to the force *K* acting along the connecting rod axis loads the main bearings of the shaft. The force *K″* may be resolved into two components: *Q′* perpendicular to the cylinder axis and *P′* acting along it.

The forces *Q′* and *Q* form a tilting moment M_{tilt}:

$$M_{tilt} = -hQ' = -hP\tan\beta = -P\tan\beta\frac{\sin(\varphi+\beta)}{\sin\beta}R = -TR = -T$$

In a multi-cylinder engine there must be a sequence in which the powers troke of each cylinder takes place, one after another. In order to evaluate the net engine torque, information on the firing order must be available. Successive firings cause a continuous torque delivery to the crankshaft output. Since the torque generated by every individual cylinder is dependent on the crank angle, the resultant engine torque is a combination of all individual torques from all cylinders and can be determined in the following form:

$$T_{\sum} = \sum_{1}^{n} T_i(\varphi_1, \Delta_{Ci}, \Delta_{Si}) \qquad (12)$$

where: φ_1 the crank angle of first cylinder; Δ_{Ci} is the crank angle of *i*th cylinder relative to the crank of cylinder 1; Δ_{Si} is the angle of rotation of the crankshaft for the current state of the *i*th cylinder;

RESULTS AND DISCUSSION

The analysis of kinematics and dynamics of the crank mechanism is normally carried out by graphical method [1,2]. This section presents some simulation results that were generated using models and equations that have been outlined. In order to study kinematics and dynamics of crank mechanism, MATLAB software is used with a set of parameters of IEC [1]: *m_p= 430 g; m_{rod} =440 g; l= 140 mm; R=49 mm; piston area 5800 mm^2*

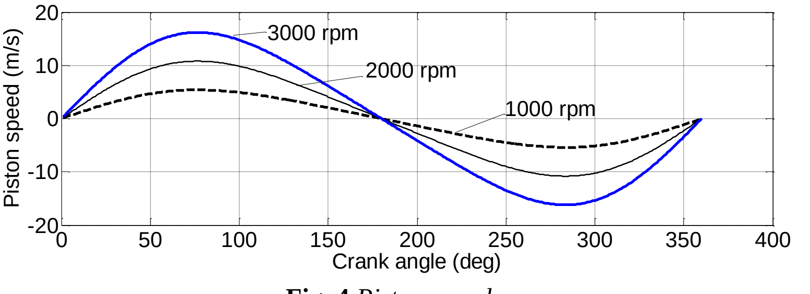

Fig. 4 *Piston speed*

Results for the piston speed are plotted in Figures 4. Positive values refer to the downward direction and negative values to the upward direction.

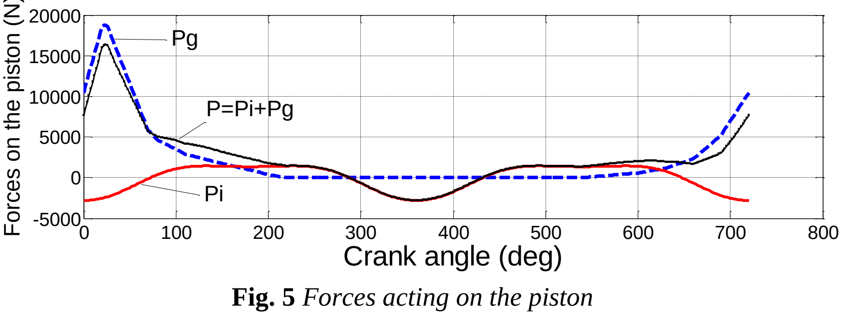

Fig. 5 *Forces acting on the piston*
P – Total force
Pi – inertia force
Pg- Gas force

The total force curve $P = f(\varphi)$ in Fig. 5 shows that at the end of the compression stroke and the beginning of the power stroke the forces of inertia reduce the effort produced by gas pressure on the piston. The total force acting on the piston is important for the further calculations and the power source of the engine.

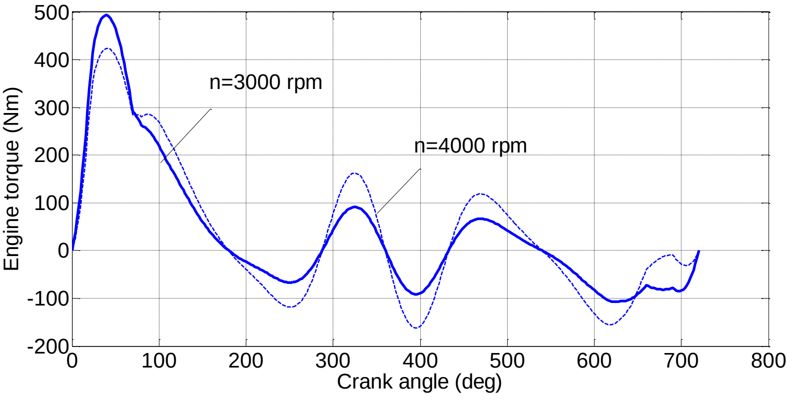

Fig. 6a *Single cylinder Engine Torque at speed of crank shaft 3000 rpm and 4000 rpm*

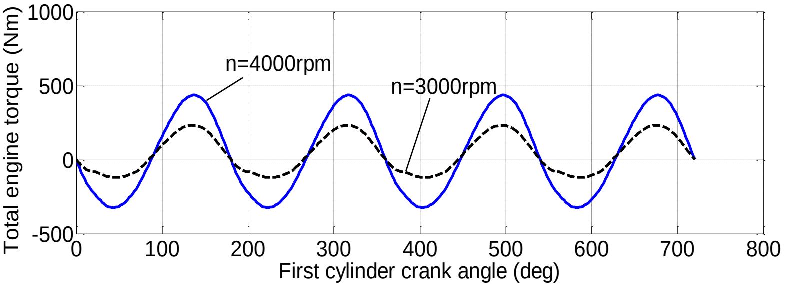

Fig.6b. *Four cylinder engine torque at speed of crank shaft 3000 rpm and 4000 rpm*

In Fig. 6a presents the torque of a single cylinder engine and in Fig. 6b presents the total torque of a four-cylinder engine with 1–3–4-2 firing order. From Fig. 6a and Fig.6b, it can be seen that the differences in the overall smoothness of torque delivery of a single and four- cylinder engine.

CONCLUSIONS

The analysis of kinematics and dynamics of the crank mechanism is normally carried out by graphical method. The study aims to conduct the kinematic and dynamic simulation of the four stroke engine in MATLAB software. The study results showed that the use of MATLAB software allows detailed simulation and dynamical structural dynamics of the crank mechanism and allow to save calculation time. The research results obtained using this software suite with several published works.

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TÓM TẮT

MÔ HÌNH HÓA VÀ MÔ PHỎNG ĐỘNG HỌC ĐỘNG LỰC HỌC CƠ CẤU THANH TRUYỀN TRỤC KHUỶU ĐỘNG CƠ Ô TÔ

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Nghiên cứu này nhằm tiến hành mô phỏng động học và động lực học của động cơ bốn kỳ bằng phần mềm MATLAB. Một phương pháp mô hình hóa cho phép tính toán động học và động lực học của cơ cấu thanh truyền trục khuỷu của động cơ ô tô đã được xây dựng. Các kết quả nghiên cứu về động học và động lực học của cơ cấu thanh truyền truyền trục khuỷu động cơ 4 kỳ cũng được trình bày chi tiết trong bài báo.

Từ khóa: *động học, động lực học, cơ cấu thanh truyền trục khuỷu, mô phỏng*

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