

## A NUMERIC METHOD TO DETERMINE WORKSPACE OF INDUSTRIAL ROBOTS

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### SUMMARY

Shape and capacity of robotic workspace are critical information when selecting robot for particular purpose. This paper presents a numeric method to determine workspace of any dumb robot. This method is the consequence of the application of GRG algorithm when transforming the robot kinematic problem into optimization combined with the bisection method. The shape and capacity of robot workspace resulted from the method in 3D format with adjustable accuracy can be chosen. This results can be used in robot designing.

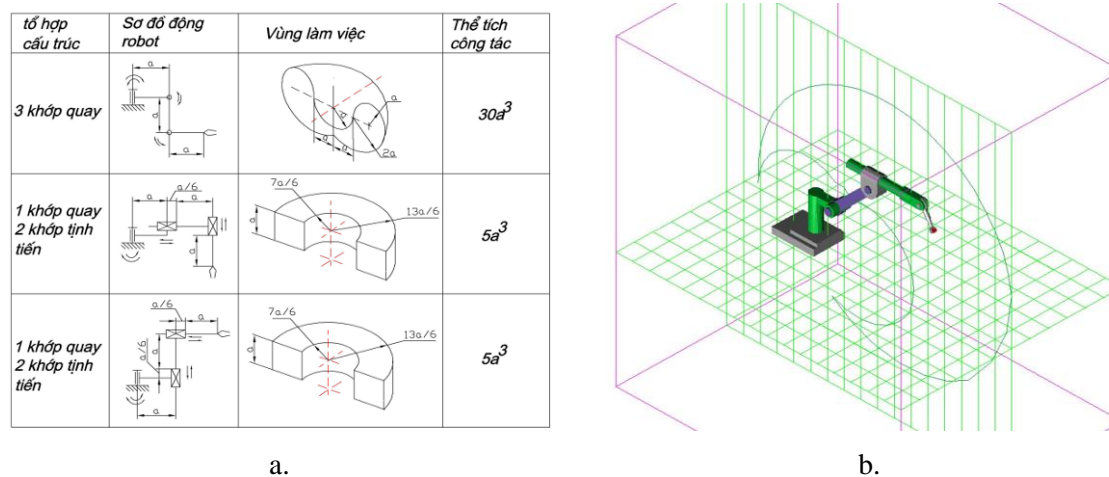
**Key words:** robot workspace, numeric method, grg algorithm, bisection method, robot designing

### ROBOT WORKSPACE

Robot workspace is the movement field of the final activator. This is a continuous space with particular shape and capacity. The determination of this space is not so difficult in flat or simple robots. However, in parallel or serial robots with 6 degrees of freedom, the inference is not simple.

Workspace can be defined in two ways:

- The zone in which the final activator can reach and direct the tool (Type I).
- The pure reachable zone (Type II).



**Figure 1.** Workspace by compounding the basic geometric shapes for each joint (a) and workspace in front view 2D (b)

Workspace type II always contains workspace type I as the strict requirements of the Type I eliminated a large number of points which are not satisfied the tooling orientation. The description of the two types in detail helps to build boundary conditions to find the shape and size of workspace.

In fact, in the catalogs provided by robot manufacturers, workspace type II is presented in front view and top view without 3D view. In this paper, the determination of both types in 3D view is presented.

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### ALGORITHM TO DETERMINE WORKSPACE

If we use the definition of workspace as the reachable space of final actuator then the equivalent technical interpretation is the inverse kinematic problem which must have its solution at the point where the final actuator is reachable. To actualize this, the following steps need to be done:

- Meshing the whole robot space using such rule that is easy to investigate and coordinate points in motion field.
- In each simple investigation line, in both increase and decrease direction, the boundary between the root-point and non root-point is required to point out. The middle point of these points is considered belong to limit surface with the fine grid.
- Scanning all points on the edge of investigating space to show the clear boundary between workspace and the remaining.
- With the workspace type II, the condition for a point  $p_i$  considered belonging to the workspace is the kinematic equation at that point has root.

$$f(q_1, \dots, q_n) = p_i$$

$$q_{i\min} \leq q_i \leq q_{i\max} \quad (1)$$

$$i = 1 \div n$$

In the equation above, the inequality constraint presents the mechanical structure condition. However, the kinematic equation does not present the orientation of the actuator.

With the workspace type I, mathematical model has constraints describing the orientation of the actuator basing on the particular conditions of the problem. Beside that, constraints describing mechanical structure of the robot is still the natural constraints.

$$f(q_1, \dots, q_m) = p_i$$

$$q_{i\min} \leq q_i \leq q_{i\max}$$

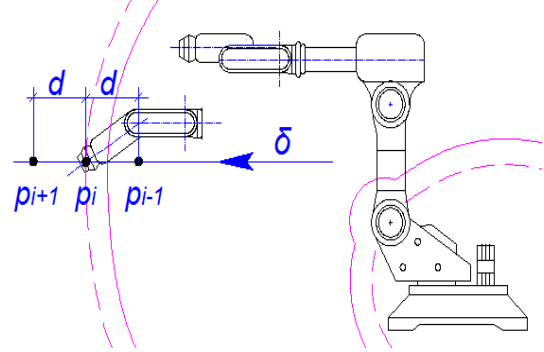
$$q_{j\min} \leq q_j \leq q_{j\max} \quad (2)$$

$$i = 1 \div n$$

$$j = n + 1 \div m$$

The orientation constraints in the model restricted the workspace. Due to this reason, the trajectory problems should be simulation checked before applying on robot as robot may not be able to move its final actuator through a hole in the workspace, at which the Jacobian matrix becomes zero.

To show a point belong to the boundary on workspace, let look closer to schema in figure 2.



**Figure 2.** Describing the boundary of workspace of robot

As can be seen from figure 2, with the moving orientation through  $p_{i-1}$ ,  $p_i$ ,  $p_{i+1}$  points, searching space  $2d$ , the problem has root at  $p_{i-1}$ . Continue searching to point  $p_{i+1}$ , the problem has no root. With  $d$  small enough, it can be approximately considered  $p_i$  which is a middle point of  $p_{i-1}p_{i+1}$  belong to the boundary of workspace. To increase the accuracy of the algorithm, the roots of equation (1) at  $p_i$  can be checked to determine either  $p_{i-1}p_i$  or  $p_i p_{i+1}$  contains point belonging to the boundary of workspace.

The searching result is complete when the algorithm is completely done in all searching directions. Set of boundary points describes the form and space of the workspace.

If the searching mesh is not done at the beginning, the bisection method can be done as alternative. Searching process stops when accuracy conditions of the results are satisfied equation (3)

$$d \geq p_{i+1} - p_i \quad (3)$$

In (3),  $p_i$  and  $p_{i+1}$  are two points appearing in consecutive searching round. One of these is belong to workspace and the other is not, equivalent to be a root or non-root of equation (1), respectively.



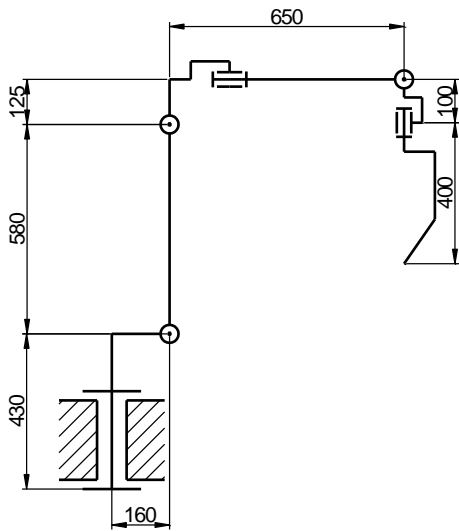
$$\begin{cases} L = f(q_1, q_2, \dots, q_n) \rightarrow \min \\ q_i \in D; \\ i = 1 \div n \end{cases} \quad (6)$$

The solution of (6) must be the root of (5). Therefore, the objective function of (6) is described as  $L = (s_x - a_{12})^2 + (a_x - a_{13})^2 + (a_y - a_{23})^2 + (p_x - a_{14})^2 + (p_y - a_{24})^2 + (p_z - a_{34})^2$  (7)

Problem (6) is stably solved using GRG algorithm with high accuracy [1]. This method is suitable for technical problems on a great scale.

**NUMERIC SIMULATION**

Considering a robot describing in figure 5, to determine the 3D workspace of this robot, the following parameters need to be examined:



*Three orientation components*

$$\begin{aligned} a_{12} &= -((C_1.C_2.C_3 - C_1.S_2.S_3).C_4 + S_1.S_4).C_5 + (-C_1.C_2.S_3 - C_1.S_2.C_3).S_5.S_6 + (-C_1.C_2.C_3 - C_1.S_2.S_3).S_4 + S_1.C_4).C_6 \\ a_{13} &= ((C_1.C_2.C_3 - C_1.S_2.S_3).C_4 + S_1.S_4).S_5 - (-C_1.C_2.S_3 - C_1.S_2.C_3).C_5 \\ a_{23} &= ((S_1.C_2.C_3 - S_1.S_2.S_3).C_4 - C_1.S_4).S_5 - (-S_1.C_2.S_3 - S_1.S_2.C_3).C_5 \end{aligned}$$

*Three position components*

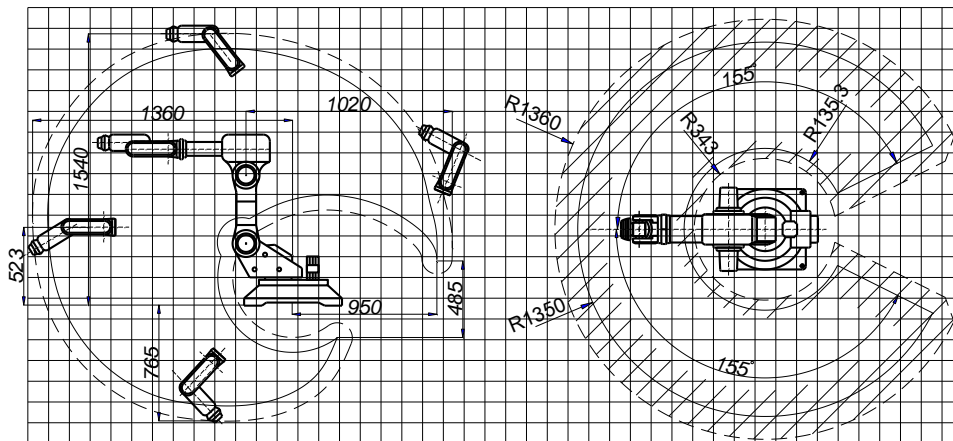
$$\begin{aligned} a_{14} &= 500.((C_1.C_2.C_3 - C_1.S_2.S_3).C_4 + S_1.S_4).S_5 - 500.(-C_1.C_2.S_3 - C_1.S_2.C_3).C_5 + 650.C_1.C_2.S_3 + 650.C_1.S_2.C_3 + 125.C_1.C_2.C_3 - 125.C_1.S_2.S_3 + 580.C_1.C_2 + 160.C_1 \\ a_{24} &= 500.((S_1.C_2.C_3 - S_1.S_2.S_3).C_4 - C_1.S_4).S_5 - 500.(-S_1.C_2.S_3 - S_1.S_2.C_3).C_5 + 650.S_1.C_2.S_3 + 650.S_1.S_2.C_3 + 125.S_1.C_2.C_3 - 125.S_1.S_2.S_3 + 580.S_1.C_2 + 160.S_1 \\ a_{34} &= 430 + 500.(S_2.C_3 + C_2.S_3).C_4.S_5 - 500.(-S_2.S_3 + C_2.C_3).C_5 + 650.S_2.S_3 - 650.C_2.C_3 + 125.S_2.C_3 + 125.C_2.S_3 + 580.S_2 \end{aligned}$$

**Figure 5.** Robot VR006-CII and its feature kinematic equation set

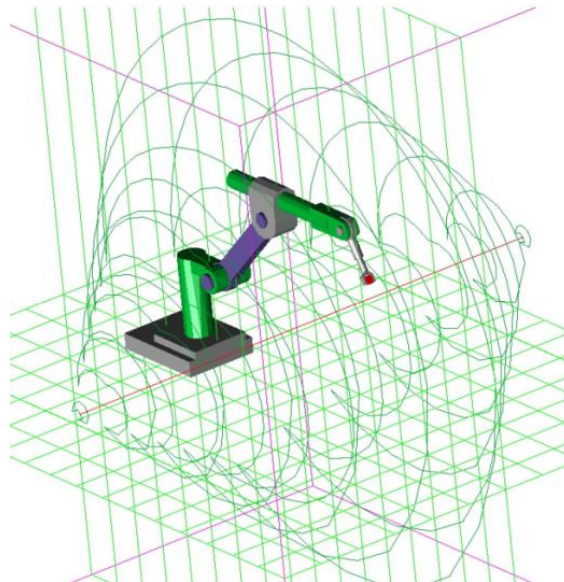
Item		VR-005CII	VR-006CII
Type		YA-11KMR51	YA-1KMR61
Degree of freedom		6 axes	
Max. allowable loading weight		5kg	6kg
Operation range	Arm	Rotation (RT) (Referenced from front)	±155°
		Upper arm (UA) (Referenced from vertical)	+150° -90°
		Front arm (FA) (Referenced from horizontal)	+180° -180° +90° -80°
	Wrist	Rotating (RW)	±180°
		Bending (BW) (Referenced from fore arm)	+95° -155°
		Twisting (TW)	±400°

**Figure 6.** Boundary conditions for movement when finding workspace of VR006 CII

Meshing the space with grid length of 50 mm, using bisect method to have smaller grids at the boundary, the results of numeric investigation of workspace type II in top view and front view are shown in figure 7.



**Figure 7.** Describing of workspace when connecting points in boundary of robot VR006- CII



**Figure 8.** Describing of workspace in 3D of robot VR006- CII.

## CONCLUSION

The combination of the bisection algorithm and GRG algorithm makes the determination of workspace of robot VR006-CII easy and effective. The expansion of this method into any dumb robots depends only on the ability to solve its feature kinematic equation set as described in (1) or (2). This ability to solve the set is proved in [1,2].

The algorithm is especially effective when being applied in parallel robots with high complexity in structure due to the limitation in imagination. In this situation, it is difficult for other methods to find workspace to be applied.

The method is especially suitable to construct 3D workspace. Therefore this method can be used as a critical part in designing and testing robot before manufacturing or operating.

## REFERENCES

1. Phạm Thành Long, A New Method to Solve the Reverse Kinematic Robot Problem, ISTS Swisotel Le Concorde, Bangkok Thailand, pp. 43-46, November 21-24/2012.
2. Li Wei Guang, TrangThanhTrung, Phạm Thành Long, A New Method to Solve the Kinematic Problem of Parallel Robots Using an Equivalent Structure, International Conference on Mechatronics and Automation Science (ICMAS 2015) Paris, France, April 13 - 14, 2015.

3 Gosselin, C., 1990, “Determination of the Work space of 6 –DOF Parallel Manipulators,” ASMEJ. Mech. Des.,112,pp.331–336.

4 Merlet, J. P., 1999, “Determination of 6D Workspaces of Gough-Type Parallel Manipulator and Comparison Between Different Geometries,” Int. J. Robot. Res.,189, pp. 902–916.

5 Pernkopf, F., and Husty, M., 2006, “Workspace Analysis of Stewart–GoughType Parallel Manipulators,” Proc. Inst. Mech. Eng., Part C: J. Mech. Eng. Sci., 2207, pp. 1019–1032.

6 Merlet, J. P., 1994, “Trajectory Verification in the Workspace for Parallel Manipulator,” Int. J. Robot. Res., 134 , pp. 326–333.

## TÓM TẮT

### MỘT PHƯƠNG PHÁP SỐ XÁC ĐỊNH VÙNG LÀM VIỆC CỦA ROBOT CÔNG NGHIỆP

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Hình dáng và thể tích vùng làm việc của robot là thông tin quan trọng khi lựa chọn ứng dụng vào các mục đích cụ thể. Bài báo này giới thiệu một phương pháp số, giúp xác định vùng làm việc của bất kỳ robot nào không tự hành. Phương pháp giới thiệu ở đây là hệ quả của việc ứng dụng phương pháp General Reduce Gradient khi chuyển bài toán động học robot sang hình thức tối ưu kết hợp với phương pháp chia đôi. Kết quả đạt được là hình dáng và kích thước vùng làm việc của robot dưới dạng 3D với độ chính xác tùy chỉnh bởi người giải. Kết quả của bài toán này có thể ứng dụng vào quá trình thiết kế robot nói chung.

**Từ khóa:** *vùng làm việc của robot, phương pháp số, thuật toán rg, phương pháp chia đôi, thiết kế robot*

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