SOLUTIONS ABOUT PROBABILISTIC CHARACTERISTICS OF DISPLACEMENTS IN A STOCHASTIC TRUSS

Dương Thế Hùng[1*], Trần Việt Thắng[2], Trần Văn Sơn[3]
[1]College of Technology - TNU, [2]College of Economics and Technology - TNU
[3]Thai Nguyen College of Electromechanics and Metallurgy

SUMMARY

A conventional structure during bearing loads usually has cross-sectional areasoften changed, not always constant because of defectiveness or corrosiveness…Moreover, in the process of working, the loads acting on structures themselves change. Therefore, this paper offers solutions of a truss mentioned the change of cross-sectional areas and loads are modeled as two types of random variables. From this model, we have proposed solutions to receive the exactly probabilistic characteristics of displacements and analyzed the effects of random parameters to expectations and variances of these displacements.
Keywords: *stochastic, random, displacement, solution*

INTRODUCTION

Models in simulating real structures always play the impotant roles because models reflect the processes of their working. Random pattern is one of the models conformed to the most realistic structures. This paper will use a random model to calculate a truss subjected to stochastic loads.

In reality, while parameters in each structure always consist of uncertain variables. Some authors [1,3,4,5] in their research go towards consider that at least exist one variable to be random. During bearing loads cross-sectional bar areas of a structure often changed, not always deteministic constant because of defectiveness or corrosiveness…Moreover, in the process of working, the loads acting on structures themselves change. In this paper, we consider the cross-section areas and loads are random variables that take on positive values, and are representable as follows [1,3,4]

$$S = S^0\left(1+\varepsilon\alpha\right),\quad Q = Q^0\left(1+\varepsilon_3\beta\right) \qquad (1)$$

Where S and Q are the same as in eq(9) after. In many research results [1,3,4,5] they often consider some random variables. However, there are nothing to find out how much the behaviors of structures depend on these random variables, especially the difference in geometry and materials during using them. Then this paper will calculate and discuss the influence of random variables are cross-sectional areas and loads to the results of displacements.

RESULTS FOR A TWO -BAR TRUSS WITH STOCHASTIC CROSS -SECTIONAL AREA

Consider a simple example of a two-bar truss structure as shown in Fig.1. Both bars have the same length L and Young's moduli E, and cross-sectional area S_1 and S_2, respectively. Assume that S_1 and S_2 are independent random variables with mean S^0 and coefficient of variation r_1,r_2; Q is a random variable with mean Q^0 and coefficient of variation r_3. We also assume S_1, S_2 and Q are independent each other.

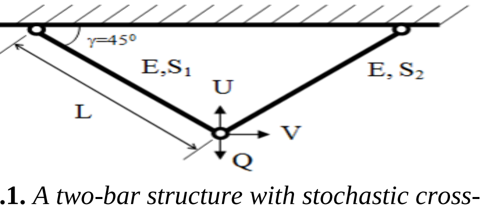

Fig.1. *A two-bar structure with stochastic cross-sectional area*

The global finite element equilibrium equation for the structure is written as

$$\frac{E}{2L}\begin{bmatrix} S_1+S_2 & -S_1+S_2 \\ -S_1+S_2 & S_1+S_2 \end{bmatrix}\begin{Bmatrix} U \\ V \end{Bmatrix} = \begin{Bmatrix} -Q \\ 0 \end{Bmatrix} \qquad (2)$$

[*] *Tel: 0982 746081, Email: hungduongxd@gmail.com*

Where $U = [U, V]^T$ is the nodal displacement vector and $F = [-Q.\ 0]^T$ is the nodal force vector. Solutions for the mean and variance of the displacements for this two-bar truss structure can be solved by computing the inverse of the stiffness matrix. The global stiffness matrix can be explicitly inverted to be

$$K^{-1} = \frac{L}{2E}\begin{bmatrix} C_1 + C_2 & -C_1 + C_2 \\ -C_1 + C_2 & C_1 + C_2 \end{bmatrix} \tag{3}$$

where $C_1=1/S_1$ and $C_2=1/S_2$ (4)

So we can be obtained the results of displacements

$$U = -\frac{L}{2E}Q(C_1 + C_2); \quad V = -\frac{L}{2E}Q(-C_1 + C_2) \tag{5}$$

The means of the displacements are obtained by applying the rule of two independent variables [2]

$$\overline{U} = -\frac{L}{2E}\overline{Q}.\left(\overline{C_1} + \overline{C_2}\right); \quad \overline{V} = -\frac{L}{2E}\overline{Q}.\left(-\overline{C_1} + \overline{C_2}\right) \tag{6}$$

where expressing bar above denote the means of the considering objects.

The variances and covariances of the displacements are

$$\begin{cases} var[U] = var\left[-\frac{L}{2E}Q\right] var[C_1 + C_2] + \\ \qquad + \left(M[C_1 + C_2]\right)^2 var\left[-\frac{L}{2E}Q\right] + \left(M\left[-\frac{L}{2E}Q\right]\right)^2 var[C_1 + C_2] \\ var[V] = var\left[-\frac{L}{2E}Q\right] var[-C_1 + C_2] + \\ \qquad + \left(M[-C_1 + C_2]\right)^2 var\left[-\frac{L}{2E}Q\right] + \left(M\left[-\frac{L}{2E}Q\right]\right)^2 var[-C_1 + C_2] \\ \mathrm{cov}[U, V] = 0 \end{cases} \tag{7}$$

here M[.] denote the mean of a variable.We can re-write the variances are following

$$\begin{cases} var[U] = \left(-\frac{L}{2E}\right)^2 \left\{var[Q].var[C_1 + C_2] + \left(\overline{[C_1 + C_2]}\right)^2 var[Q] + \left(\overline{Q}\right)^2 var[C_1 + C_2]\right\} \\ var[V] = \left(-\frac{L}{2E}\right)^2 \left\{var[Q].var[-C_1 + C_2] + \left(\overline{[-C_1 + C_2]}\right)^2 var[Q] + \left(\overline{Q}\right)^2 var[-C_1 + C_2]\right\}, \end{cases} \tag{8}$$

We can express random variables as following [3,4]

$$S_1 = S^0(1 + \varepsilon_1\alpha_1), \quad S_2 = S^0(1 + \varepsilon_2\alpha_2), \quad Q = Q^0(1 + \varepsilon_3\beta) \tag{9}$$

where we consider that S_1 and S_2 are random variables with mean S^0 and coefficient of variation r_1, r_2; Q is a random variable with mean Q^0 and coefficient of variation r_3; $\varepsilon_1, \varepsilon_2, \varepsilon_3$- is deterministic constant, $0<\varepsilon_1,\varepsilon_2<<1$, $0 \le \varepsilon_3 << 1$;

We assume α_1, α_2 is a random variable posses a uniformly distributed density function in the interval [-1,1] and β is a random variable posses a triangular distributed density function in the interval [-1,1], since the probability density $f(\alpha)$ and $f(\beta)$ of α, β, respectively equals

$$f(\alpha_1) = \begin{cases} 1/2, & |\alpha_1| \le 1 \\ 0, & else, \end{cases} \quad f(\alpha_2) = \begin{cases} 1/2, & |\alpha_2| \le 1 \\ 0, & else, \end{cases} \quad f(\beta) = \begin{cases} (1 - |\beta|), & |\beta| \le 1, \\ 0, & else, \end{cases} \tag{10}$$

We have the means, variances of the Q and S_1, S_2

$$\bar{Q} = M\left[Q\right] = \int_{-1}^{1} Qf(\beta)d\beta = Q^0,\ M\left[Q^2\right] = \int_{-1}^{1} Q^2 f(\beta)d\beta = \frac{\left(Q^0\varepsilon_3\right)^2}{6} + \left(Q^0\right)^2 \tag{11}$$

$$var\left[Q\right] = M\left[Q^2\right] - \left(M\left[Q\right]\right)^2 = \frac{\left(Q^0\varepsilon_3\right)^2}{6}; \quad r_3 = \frac{\sqrt{\text{var}\left[Q\right]}}{M\left[Q\right]} = \frac{\varepsilon_3}{\sqrt{6}} \tag{12}$$

$$M\left[S_1\right] = \int_{-1}^{1} S_1 f(\alpha_1)d\alpha_1 = S^0, \qquad M\left[S_2\right] = S^0 \tag{13}$$

$$var\left[S_1\right] = M\left[S_1^2\right] - \left(M\left[S_1\right]\right)^2 = \frac{\left(S^0\varepsilon_1\right)^2}{3}; \quad r_1 = \frac{\sqrt{\text{var}\left[S_1\right]}}{M\left[S_1\right]} = \frac{\varepsilon_1}{\sqrt{3}} \tag{15}$$

$$var\left[S_2\right] = M\left[S_2^2\right] - \left(M\left[S_2\right]\right)^2 = \frac{\left(S^0\varepsilon_2\right)^2}{3}; \quad r_2 = \frac{\sqrt{\text{var}\left[S_2\right]}}{M\left[S_2\right]} = \frac{\varepsilon_2}{\sqrt{3}} \tag{16}$$

We compute the mean and variance of C_1 as following

$$\overline{C_1} = M\left[C_1\right] = \int_{-1}^{1} \frac{1}{S^0(1+\varepsilon_1\alpha_1)} f(\alpha_1)d\alpha_1 = \frac{1}{2S^0\varepsilon_1}\ln\frac{1+\varepsilon_1}{1-\varepsilon_1}, \quad M\left[C_1^2\right] = \frac{1}{\left(S^0\right)^2}\frac{1}{1-\varepsilon_1^2} \tag{17}$$

$$var\left[C_1\right] = M\left[C_1^2\right] - \left(M\left[C_1\right]\right)^2 = \frac{1}{\left(S^0\right)^2}\left[\frac{1}{1-\varepsilon_1^2} - \left(\frac{1}{2\varepsilon_1}\ln\frac{1+\varepsilon_1}{1-\varepsilon_1}\right)^2\right] \tag{18}$$

Analogously, we also are obtained the mean and variance of C_2

$$\overline{C_2} = M\left[C_2\right] = \int_{-1}^{1} \frac{1}{S^0(1+\varepsilon_2\alpha_2)} f(\alpha_2)d\alpha_2 = \frac{1}{2S^0\varepsilon_2}\ln\frac{1+\varepsilon_2}{1-\varepsilon_2} \tag{19}$$

$$var\left[C_2\right] = \frac{1}{\left(S^0\right)^2}\left[\frac{1}{1-\varepsilon_2^2} - \left(\frac{1}{2\varepsilon_2}\ln\frac{1+\varepsilon_2}{1-\varepsilon_2}\right)^2\right] \tag{20}$$

Because S_1 and S_2 are independent therefore C_1 and C_2 are also independent, so that we are obtained the means and variances of $(C_1 + C_2)$, $(-C_1 + C_2)$ as following

$$\overline{C_1 + C_1} = M\left[C_1\right] + M\left[C_2\right] = \frac{1}{2S^0}\left(\frac{1}{\varepsilon_1}\ln\frac{1+\varepsilon_1}{1-\varepsilon_1} + \frac{1}{\varepsilon_2}\ln\frac{1+\varepsilon_2}{1-\varepsilon_2}\right) \tag{21}$$

$$\overline{-C_1 + C_1} = M\left[-C_1\right] + M\left[C_2\right] = \frac{1}{2S^0}\left(-\frac{1}{\varepsilon_1}\ln\frac{1+\varepsilon_1}{1-\varepsilon_1} + \frac{1}{\varepsilon_2}\ln\frac{1+\varepsilon_2}{1-\varepsilon_2}\right) \tag{22}$$

$$\begin{aligned} var\left[C_1 + C_2\right] &= var\left[-C_1 + C_2\right] = var\left[C_1\right] + var\left[C_2\right] = \\ &= \frac{1}{\left(S^0\right)^2}\left[\frac{1}{1-\varepsilon_1^2} + \frac{1}{1-\varepsilon_2^2} - \left(\frac{1}{2\varepsilon_1}\ln\frac{1+\varepsilon_1}{1-\varepsilon_1}\right)^2 - \left(\frac{1}{2\varepsilon_2}\ln\frac{1+\varepsilon_2}{1-\varepsilon_2}\right)^2\right] \end{aligned} \tag{23}$$

Substituting eqns (11),(12) and eqns (21),(22),(23) into eqns (6),(8) yield the means and variances of displacements U,V

$$\overline{U} = -\frac{L}{E}\frac{Q^0}{2S^0}\phi_U; \qquad \overline{V} = -\frac{L}{E}\frac{Q^0}{2S^0}\phi_V \tag{24}$$

$$\mathrm{var}\left[U\right]=\left(-\frac{LQ^{0}}{2ES^{0}}\right)^{2}\varphi_{U}\,;\qquad \mathrm{var}\left[V\right]=\left(-\frac{LQ^{0}}{2ES^{0}}\right)^{2}\varphi_{V} \tag{25}$$

where $$\phi_{U}=\frac{1}{2}\left(\frac{1}{\varepsilon_{1}}\ln\frac{1+\varepsilon_{1}}{1-\varepsilon_{1}}+\frac{1}{\varepsilon_{2}}\ln\frac{1+\varepsilon_{2}}{1-\varepsilon_{2}}\right);\quad \phi_{V}=\frac{1}{2}\left(-\frac{1}{\varepsilon_{1}}\ln\frac{1+\varepsilon_{1}}{1-\varepsilon_{1}}+\frac{1}{\varepsilon_{2}}\ln\frac{1+\varepsilon_{2}}{1-\varepsilon_{2}}\right) \tag{26}$$

$$\varphi_{U}=\left\{\begin{array}{l}\left[\frac{(\varepsilon_{3})^{2}}{6}+1\right]\left[\frac{1}{1-\varepsilon_{1}^{2}}+\frac{1}{1-\varepsilon_{2}^{2}}-\left(\frac{1}{2\varepsilon_{1}}\ln\frac{1+\varepsilon_{1}}{1-\varepsilon_{1}}\right)^{2}-\left(\frac{1}{2\varepsilon_{2}}\ln\frac{1+\varepsilon_{2}}{1-\varepsilon_{2}}\right)^{2}\right]+\\+\frac{(\varepsilon_{3})^{2}}{6}\left[\frac{1}{2}\left(\frac{1}{\varepsilon_{1}}\ln\frac{1+\varepsilon_{1}}{1-\varepsilon_{1}}+\frac{1}{\varepsilon_{2}}\ln\frac{1+\varepsilon_{2}}{1-\varepsilon_{2}}\right)\right]^{2}\end{array}\right\} \tag{27}$$

$$\varphi_{V}=\left\{\begin{array}{l}\left[\frac{(\varepsilon_{3})^{2}}{6}+1\right]\left[\frac{1}{1-\varepsilon_{1}^{2}}+\frac{1}{1-\varepsilon_{2}^{2}}-\left(\frac{1}{2\varepsilon_{1}}\ln\frac{1+\varepsilon_{1}}{1-\varepsilon_{1}}\right)^{2}-\left(\frac{1}{2\varepsilon_{2}}\ln\frac{1+\varepsilon_{2}}{1-\varepsilon_{2}}\right)^{2}\right]+\\+\frac{(\varepsilon_{3})^{2}}{6}\left[\frac{1}{2}\left(-\frac{1}{\varepsilon_{1}}\ln\frac{1+\varepsilon_{1}}{1-\varepsilon_{1}}+\frac{1}{\varepsilon_{2}}\ln\frac{1+\varepsilon_{2}}{1-\varepsilon_{2}}\right)\right]^{2}+\end{array}\right\} \tag{28}$$

Noting that if $\varepsilon_1,\varepsilon_2,\varepsilon_3\to 0$ then $\lim \phi_U=2$, $\lim \phi_V=0$, $\lim \varphi_U=0$ and $\lim \varphi_V=0$ (29)

And then we yield coefficients of variations of U and V

$$c.o.v\left(U\right)=\frac{\sqrt{\mathrm{var}[U]}}{\bar{U}}=\frac{\sqrt{\varphi_{U}}}{\phi_{U}},\qquad c.o.v\left(V\right)=\frac{\sqrt{\mathrm{var}[V]}}{\bar{V}}=\frac{\sqrt{\varphi_{V}}}{\phi_{V}} \tag{30}$$

Discussions of displacement results

From eqns (24),(26) shown explicitly that the mean values of U,V did not consist of ε_3, so that they only denpend on $\varepsilon_1,\varepsilon_2$ but not on ε_3.

From eqns(25),(27),(28) it is clearly seen that the quantities $\phi_U,\phi_V,\varphi_U,\varphi_V$ affect the mean and variance values of U,V. Hence, we can consider the values of $\phi_U,\phi_V,\varphi_U,\varphi_V$ will be enough to evaluate the amplitudes of U.V. By assigning the value of ε_1 that is changed from 0.01 to 0.6, ε_2 changed from 0.01 to 0.3 and ε_3 changed from 0 to 0.3 obtained the value of ϕ_U,ϕ_V,φ_Ushown in Fig. 2,3,4.

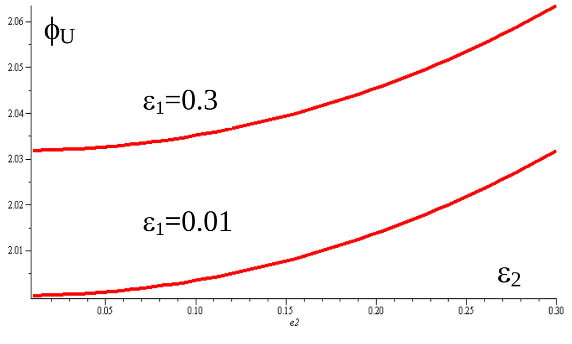

Fig. 2. *The results of ϕ_U are changing depended on values of ε_1*

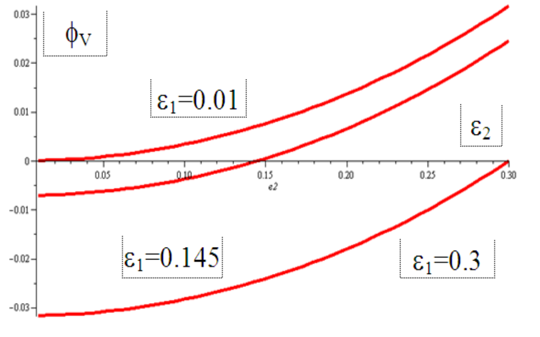

Fig. 3. *The results of ϕ_V are changing depended on values of ε_1*

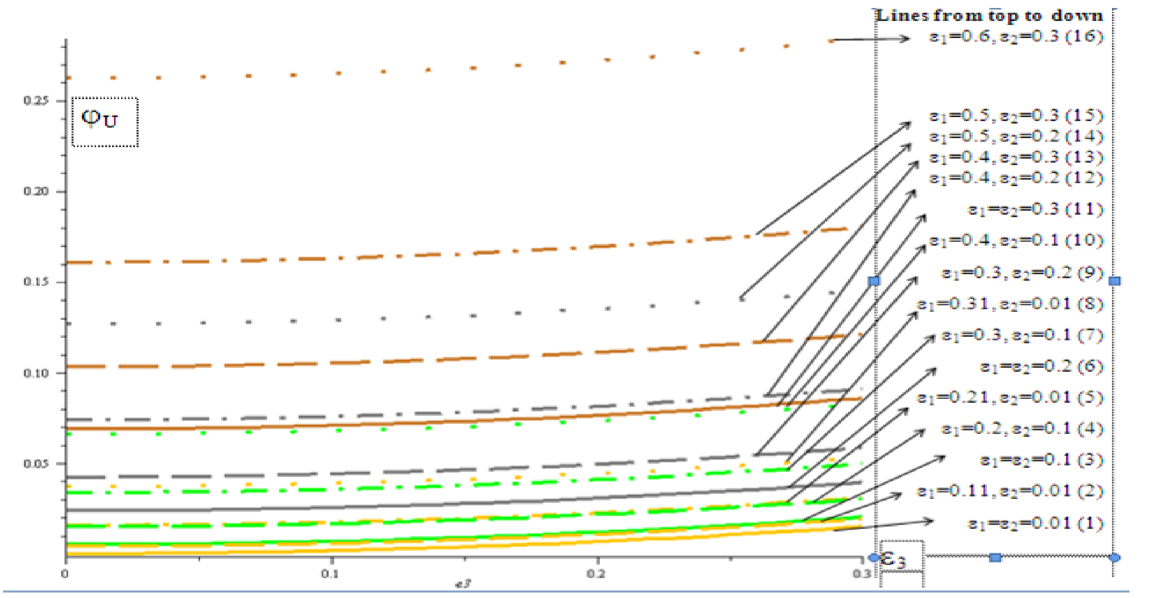

Fig. 4. *The results of φ_U are changing depended on values of $\varepsilon_1,\varepsilon_2,\varepsilon_3$*

Table 1

Type of steel	ϕ14	ϕ13.5	ϕ13	ϕ12.5	ϕ11.2	ϕ11
Cross-sectional area (mm2)	153.9	143.1	132.7	122.7	98.5	95.0
ε	0.30					
r	0.175					

Table 2

Type of steel	ϕ14	ϕ13.8	ϕ13.6	ϕ13.4	ϕ13.2	ϕ13.0
Cross-sectional area (mm2)	153.9	149.6	145.3	141.0	136.8	132.7
ε	0.09					
r	0.051					

Let's consider on Fig.4 that the values of φ_U depend on different between $\varepsilon_1,\varepsilon_2$ and ε_3 with 16 cases of changing. If ε_1 and ε_2 are less different (in cases of $\varepsilon_1 \approx \varepsilon_2$) then the values of φ_U have small amplitudes when we compare with cases of ε_1 and ε_2 to be equivalent. Consequently, that the responsible values depend on the difference ofthe other elements are enormous. In other words, during the process of working, if two bars in the truss have been corroded differently then displacements in the structures will be increasing.

In order to determine the values of ε_1 and ε_2 we can know that for long time two bars have been corroded, in Table 1 there are the discrete statistical values of cross-sectional area of left bar (S_1) and in Table 2 are of the right bar (S_2). We will calculate ε_{b1}=0.30 and ε_{b2}=0.09. From this two tables to reveal that if a cross-sectional area is changed more and more largely we will have coefficients of variations *r* (i.e ε) to be larger.

CONCLUSIONS

This paper has calculated and received the results of the means and variances of displacements in equations (24),(25) – those are new results of proposal solutions. These exact results are combined by two types of random variables. This is extremely important significance because the calculation model is closer to a real structural model. In special cases (according to eq (29)), if $\varepsilon_1=\varepsilon_2=\varepsilon_3=0$ we could get again the results of deterministic solutions.

The means of displacements in eqns (24) did not depend on the part of random loads, therefore during calculation we could ignore this part. However, the variances depend on both random loads and random materials (cross-sectional areas). The range of variation depends on the values of *r* – the coefficient of variation (i.e ε) assigned larger or smaller.

Following above reviews, during the time, in a conventional structurethe size and levels about geometry or materials usually have changed. From which engineers must consider precisely to choose solutions during their designing, examination and reliability evaluation.

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TÓM TẮT

KẾT QUẢ TÍNH TOÁN ĐẶC TRƯNG XÁC SUẤT CỦA CHUYỂN VỊ TRONG HỆ GIÀN CÓ THAM SỐ NGẪU NHIÊN

Dương Thế Hùng[1*], Trần Việt Thắng[2], Trần Văn Sơn[3]
[1]Trường Đại học Kỹ thuật Công nghiệp – ĐH Thái Nguyên
[2]Trường Cao đẳng kinh tế kỹ thuật – ĐH Thái Nguyên
[3]Trường Cao đẳng cơ điện luyện kim Thái Nguyên

Một kết cấu trong quá trình chịu lực, diện tích tiết diện thanh thường thay đổi, không phải lúc nào cũng là hằng số vì theo thời gian tiết diện thay đổi do khuyết tật hoặc ăn mòn…Hơn nữa, trong quá trình chịu lực, bản thân tải trọng cũng thay đổi. Vì vậy, bài báo đưa ra mô hình tính toán hệ giàn kể đến sự thay đổi của diện tích tiết diện và tải trọng được mô hình hóa là hai biến ngẫu nhiên. Từ mô hình đó, đã nhận được kết quả tính toán chính xác đặc trưng xác suất của chuyển vị và phân tích ảnh hưởng của tham số ngẫu nhiên đến kỳ vọng và phương sai của chuyển vị.

Từ khóa: *ngẫu nhiên, chuyển vị, dầm, lời giải*

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Phản biện khoa học: *PGS.TS Ngô Như Khoa - Trường Đại học Kỹ thuật Công nghiệp - ĐHTN*

[*] *Tel: 0982 746081, Email: hungduongxd@gmail.com*