NATURAL FREQUENCY OF FLUID -FILLED LAMINATED COMPOSITE CYLINDRICAL SHELLS ON ELASTIC FOUNDATIONS

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SUMMARY

In this paper, natural frequency of completely fluid-filled composite circular cylindrical shells on Winkler and Pasternak elastic foundations are studied. The Dynamic Stiffness Method is employed to solve the cylindrical shell problem. Natural frequencies of fluid-filled cylindrical shells based on elastic foundations are evaluated. It is observed that frequencies are strongly affected when a cylindrical shell is attached with elastic foundations. This analysis can be extended to investigate the other aspects like buckling and dynamic response involving different types of materials used for cylindrical shells.

Keywords: *Natural frequency, Fluid-filled composite cylindrical shells, Dynamic Stiffness Method, Winkler and Pasternak Elastic foundations*

INTRODUCTION

Fluid-filled composite circular cylindrical shells on Winkler and Pasternak elastic foundations are popular structures in engineering applications including aeroplanes, ships and construction buildings. Lots of research, including theoretical, numerical and experimental studies have been carried out to investigate the dynamic performance of shells with different shapes and boundary conditions. Free vibration of a partially fluid-filled cross-ply laminated composite circular cylindrical shell is investigated by Xi et al. [1, 2] using a semi-analytical finite element technique based on the Reissner–Mindlin theory and compressible fluid equations. Vibration analysis of thick axis-symmetric laminated composite shells on Winkler elastic foundation by Continuous Element Method was studied by Nguyen Manh Cuong, Tran Ich Thinh et al. [3]. The vibration analysis of laminated orthotropic shells with different boudary conditions and resting on elastic foundation was conducted by Sofiyev et al. [4]. Although some studied focusing on different aspects the laminated composite structures have been reported, free vibration investigation of fluid-filled composite circular cylindrical shells based on Winkler and Pasternak elastic foundations is still absent.

This paper presents a detailed study of free vibration of the fluid-filled composite circular cylindrical shells on Winkler and Pasternak elastic foundations. The Dynamic Stiffness Method is used to solve the cylindrical shell problem. Natural frequencies of fluid-filled cylindrical shells based on elastic foundations are evaluated. Illustrative examples are provided to demonstrate the accuracy and efficiency of the developed numerical procedure.

FORMULATION OF CROSS -PLY LAMINATED COMPOSITE CIRCULAR CYLINDRICAL SHELLS WITH FLUID BASED ON ELASTIC FOUNDATIONS

Displacements, forces and moment resultants of cylindrical shells

Consider a thick circular cylindrical shell of length L , thickness h and radius R . The shell consists of a finite number of layers which are perfectly bonded together. Following Reissner-Mindlin assumption, the displacement components are assumed to be:

$$u(x,\theta,z,t) = u_0(x,\theta,t) + z\varphi_x(x,\theta,t); v(x,\theta,z,t) = v_0(x,\theta,t) + z\varphi_\theta(x,\theta,t); w(x,\theta,z,t) = w_0(x,\theta,t)$$

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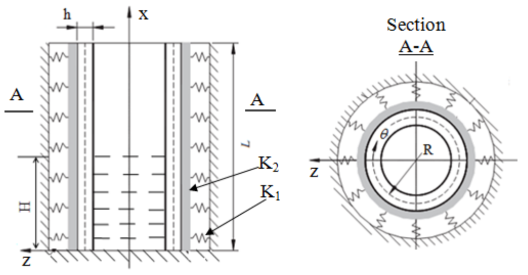

Figure 1. *Laminated composite cylindrical shell fluid on elastic foundations*

where u_0 and v_0 are the in-plane displacements of the shell in the mid-plane, and φ_x and φ_θ are the shear rotations of any point on the middle surface of the shell. The strain-displacement relations of cylindrical shell of radius R can be written as:

$$\varepsilon_x = \frac{\partial u_0}{\partial x} + z\frac{\partial \varphi_x}{\partial x};\ \varepsilon_\theta = \frac{1}{R}\frac{\partial v_0}{\partial \theta} + \frac{z}{R}\frac{\partial \varphi_\theta}{\partial \theta} + \frac{w_0}{R};$$

$$\gamma_{x\theta} = \frac{1}{R}\frac{\partial u_0}{\partial \theta} + \frac{\partial v_0}{\partial x} + z\left(\frac{1}{R}\frac{\partial \varphi_x}{\partial \theta} + \frac{\partial \varphi_\theta}{\partial x}\right);$$

$$\gamma_{xz} = \varphi_x + \frac{\partial w_0}{\partial x};\ \gamma_{\theta z} = \varphi_\theta + \frac{1}{R}\frac{\partial w_0}{\partial \theta} - \frac{v_0}{R} \qquad (2)$$

For general cross-ply composite laminated cylindrical shells, forces and moment resultants are determined by [7]:

$$N_x = A_{11}\frac{\partial u_0}{\partial x} + A_{12}\left(\frac{\partial v_0}{R\partial \theta} + \frac{w_0}{R}\right) + B_{11}\frac{\partial \varphi_x}{\partial x} + B_{12}\frac{\partial \varphi_\theta}{R\partial \theta};\ N_\theta$$

$$= A_{12}\frac{\partial u_0}{\partial x} + A_{22}\left(\frac{\partial v_0}{R\partial \theta} + \frac{w_0}{R}\right) + B_{12}\frac{\partial \varphi_x}{\partial x} + B_{22}\frac{\partial \varphi_\theta}{R\partial \theta}$$

$$N_{x\theta} = A_{66}\left(\frac{\partial v_0}{\partial x} + \frac{\partial u_0}{R\partial \theta}\right) + B_{66}\left(\frac{\partial \varphi_\theta}{\partial x} + \frac{\partial \varphi_x}{R\partial \theta}\right);\ M_x$$

$$= B_{11}\frac{\partial u_0}{\partial x} + B_{12}\left(\frac{\partial v_0}{R\partial \theta} + \frac{w_0}{R}\right) + D_{11}\frac{\partial \varphi_x}{\partial x} + D_{12}\frac{\partial \varphi_\theta}{R\partial \theta}$$

$$M_\theta = B_{12}\frac{\partial u_0}{\partial x} + B_{22}\left(\frac{\partial v_0}{R\partial \theta} + \frac{w_0}{R}\right) + D_{12}\frac{\partial \varphi_x}{\partial x} + D_{22}\frac{\partial \varphi_\theta}{R\partial \theta};$$

$$M_{x\theta} = B_{66}\left(\frac{\partial v_0}{\partial x} + \frac{\partial u_0}{R\partial \theta}\right) + D_{66}\left(\frac{\partial \varphi_\theta}{\partial x} + \frac{\partial \varphi_x}{R\partial \theta}\right);$$

$$Q_x = kA_{55}\left(\varphi_x + \frac{\partial w_0}{\partial x}\right)$$

$$Q_\theta = kA_{44}\left(\varphi_\theta + \frac{\partial w_0}{R\partial \theta} - \frac{v_0}{R}\right) \qquad (3)$$

Where A_{ij}, B_{ij}, D_{ij} are the laminate stiffness coefficients and are defined by [7]; and k=5/6: the shear correction factor, z_{k-1} and z_k are boundaries of the k^{th} layer.

Equation of motions

The equations of motions based on the first-order shear deformation shell theory for a laminated circular cylindrical shell filled with fluid taking into account hydrodynamic pressure P and based on elastic foundations are:

$$\frac{\partial N_x}{\partial x} + \frac{1}{R}\frac{\partial}{\partial \theta}\left(N_{x\theta} - \frac{1}{2R}M_{x\theta}\right) = I_0\frac{\partial^2 u_0}{\partial t^2} + I_1\frac{\partial^2 \varphi_x}{\partial t^2}$$

$$\frac{\partial}{\partial x}\left(N_{x\theta} + \frac{1}{2R}M_{x\theta}\right) + \frac{\partial N_\theta}{R\partial \theta} + \frac{Q_\theta}{R} = I_0\frac{\partial^2 v_0}{\partial t^2} + I_1\frac{\partial^2 \varphi_\theta}{\partial t^2}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_\theta}{R\partial \theta} - \frac{N_\theta}{R} - P - K_1 w_0 + K_2\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{1}{R}\frac{\partial w_0}{\partial x} + \frac{1}{R^2}\frac{\partial^2 w_0}{\partial \theta^2}\right) = I_0\frac{\partial^2 w_0}{\partial t^2}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{x\theta}}{R\partial \theta} - Q_x = I_1\frac{\partial^2 u_0}{\partial t^2} + I_2\frac{\partial^2 \varphi_x}{\partial t^2}$$

$$\frac{\partial M_{x\theta}}{\partial x} + \frac{\partial M_\theta}{R\partial \theta} - Q_\theta = I_1\frac{\partial^2 v_0}{\partial t^2} + I_2\frac{\partial^2 \varphi_\theta}{\partial t^2} \qquad (4)$$

where:

$$I_i = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \rho^{(k)} z^i dz \qquad (i = 0,1,2)$$

in which $\rho^{(k)}$ is the material mass density of the k^{th} layer and K_1 is the Winkler foundation modulus; K_2 represents the shear modulus.

The cylindrical shell is partially or completely filled with an incompressible, inviscid liquid. For the steady-state case, the potential function Φ satisfies the Laplace equation: $\Delta\Phi = 0$

Then, the Bernoulli equation is written as: $\frac{\partial \Phi}{\partial t} + \frac{P}{\rho_f} = 0$

The condition of impermeability of the surface of shell in contact with fluid may be written as:

$$v_f = \left.\frac{\partial \Phi}{\partial r}\right|_\Sigma = \left.\frac{\partial w_0}{\partial t}\right|_\Sigma$$

where v_f is the velocity of fluid, Σ is the contact surface.

Thus [8]:

$$P = -\rho_f \frac{1}{m + k_n r I_{m+1}(k_n r)/I_m(k_n r)}\left.\frac{\partial^2 w_0}{\partial t^2}\right|_\Sigma$$

The term k_n will be determined based on the fluid boundary condition.

DYNAMIC STIFFNESS MATRIX FORMULATION

Here, the state-vector **y** = $\{u_0, v_0, w_0, \varphi_S, \varphi_\theta, N_x, N_{x\theta}, Q_x, M_x, M_{x\theta}\}^T$ and the Lévy series expansion for state variables is written as:

$$\{u_o(x,\theta,t), w_o(x,\theta,t), \phi_\theta(x,\theta,t), N_x(x,\theta,t), Q_x(x,\theta,t), M_x(x,\theta,t)\}^T$$

$$= \sum_{m=1}^{\infty}\{u_m(x), w_m(x), \phi_{\theta m}(x), N_{x_m}(x), Q_{xm}(x), M_{x_m}(s)\}^T \cos m\theta e^{i\omega t}$$

$$\{v_o(x,\theta,t), \phi_x(x,\theta,t), N_\theta(x,\theta,t), Q_\theta(x,\theta,t), M_\theta(x,\theta,t)\}^T$$

$$= \sum_{m=1}^{\infty}\{v_m(x), \phi_{xm}(x), N_{\theta m}(x), Q_{\theta m}(x), M_{\theta m}(x)\}^T \sin m\theta e^{i\omega t} \quad (7)$$

Substitute formulas (7) into equations (3) and (4), using the approach developed in the previous researches [5, 6], a system of 10 differential equations is obtained and written in the matrix form for each circumferential mode *m*:

$$\frac{d\{\mathbf{y}\}_m^T}{dx} = [\mathbf{A}]_m \{\mathbf{y}\}_m^T \quad (8)$$

The dynamic transfer matrix **[T]**$_m$ is given by : $[\mathbf{T}]_m = e^{[\mathbf{A}]_m L}$ (9)

Finally, the dynamic stiffness matrix $[\mathbf{K}(\omega)]_m$ is determined by [5,6]:

$$[\mathbf{K}(\omega)]_m = \begin{bmatrix} \mathbf{T}_{12}^{-1}\mathbf{T}_{11} & -\mathbf{T}_{12}^{-1} \\ \mathbf{T}_{21} - \mathbf{T}_{22}\mathbf{T}_{12}^{-1}\mathbf{T}_{11} & \mathbf{T}_{22}\mathbf{T}_{12}^{-1} \end{bmatrix}_m$$

The dynamic stiffness matrix can be easily assembled with other element matrices in order to model a long cylindrical shells or cylinders with portions of different properties.

The natural frequencies of the structure and harmonic responses are determined by using the procedure detailed in [5].

NUMERICAL RESULTS AND DISCUSSION

A computer program based on Matlab is developed using the CEM to solve a number of numerical examples on free vibration of composite cylindrical shells with fluid and based on elastic foundations.

The composite material of the shells has the following properties: E_1=206.9 GPa; E_2=18.62 GPa; ν_{12}=0.28, G_{12}=4.48 GPa; G_{13}=4.48 GPa; G_{23}=2.24 GPa; ρ=2048 kg/m^3; layer scheme: [0°/90°/0°/90°]. The water characteristics are: ρ_f = 1000 kg/m^3, c = 1500 m/s. Dimensions of the circular cylindrical shells: h=9.525 mm; R=0.1905 m; L=0.381 m.

The effect of both elastic foundation stiffnesses (K_1, K_2) on the first natural frequencies of wet shells are listed in Table 1 and illustrated in Fig 2.

Table 1. *Effects of foundation stiffnesses on first fundamental frequencies of wet cylindrical shells.*

K_2 \ K_1	0	10^6	1,5x10^6	2x10^6	2,5x10^6
0	419.8	428.9	433.4	437.8	442.2
10^4	429.2	438.1	442.5	446.8	451.1
1,5x10^4	433.8	442.6	446.9	451.2	455.5
2x10^4	438.8	447.1	451.4	455.6	459.8
2,5x10^4	442.8	451.5	455.7	460.0	464.1

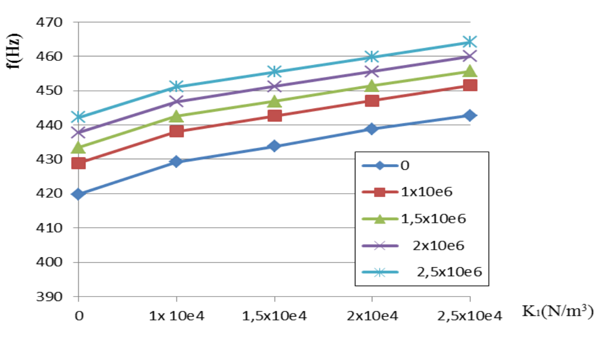

Figure 2. *Effects of foundation stiffnesses (K_1, K_2) on fundamental frequencies of wet cylindrical shells.*

The effect of ratio R/h on the first natural frequencies of wet shells are listed in Table 2 and illustrated in Fig. 3. When the ratio R/h increases, the fundamental frequencies are decreased.

Table 2. *Effect of ratio R/h on the first natural frequencies of wet [$0^o/90^o/0^o/90^o$] cylindrical shells.*

K_1, K_2	**R/h**				
	20	**40**	**60**	**80**	**100**
K_1=0, K_2=0	419.8	342.4	271.7	241.5	223.3
K_1=2.5x10^6	422.1	347.9	281.2	254.9	240.2
K_2=2x10^4	421.7	352.8	289.6	266.6	254.9
K_1=2.5x10^6, K_2=2x10^4	424	358.1	298.6	278.8	269.9

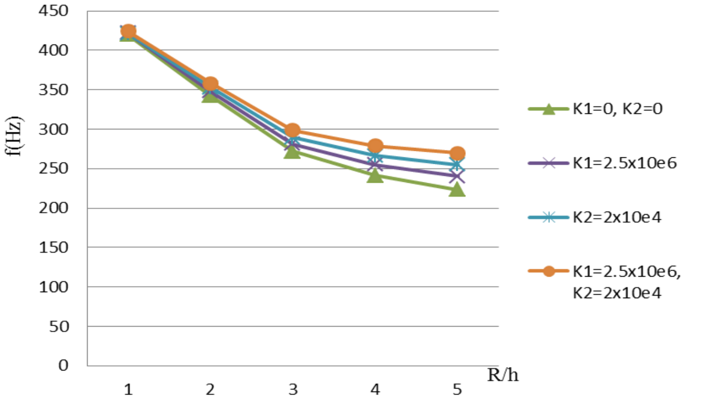

Figure 3. *Effect of ratio R/h on the first natural frequencies of wet [$0^o/90^o/0^o/90^o$] cylindrical shells*

CONCLUSIONS

In this work, vibration frequency analysis of fluid-filled laminated composite circular cylindrical shells based on elastic foundations is presented for clamped-free conditions. The Dynamic Stiffness Method is used to derive the composite shell frequency equation including the elastic foundation and fluid loading terms. The influence of elastic foundation is more pronounced on the shell frequencies. This analysis can be extended to investigate the other aspects like buckling and dynamic response involving different types of materials used for cylindrical shells.

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TÓM TẮT

DAO ĐỘNG CỦA VỎ TRỤ CHỨA CHẤT LỎNG VÀ TIẾP XÚC VỚI NỀN ĐÀN HỒI

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Bài báo này tập trung nghiên cứu dao động tự do của vỏ trụ composite lớp chứa chất lỏng đồng thời tiếp xúc với nền đàn hồi. Để giải quyết bài toán, tác giả đã đề xuất Phương pháp Độ cứng động để xác định tần số dao động riêng của vỏ trụ chứa chất lỏng nằm trên nền đàn hồi. Kết quả chỉ ra rằng tần số dao động của vỏ trụ bị ảnh hưởng mạnh khi vỏ trụ chứa chất lỏng tiếp xúc đồng thời với cả hai nền đàn hồi Winkler và Pasternak. Hướng nghiên cứu này hoàn toàn có thể mở rộng cho các trường hợp khác như ổn định và tải động của vỏ trụ làm bằng các loại vật liệu và chịu các liên kết khác nhau.

Từ khóa: *dao động tự do, vỏ trụ composite chứa chất lỏng, ma trận động cứng động, nền đàn hồi Winkler và Fasternak*

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