# COMPUTER AIDED CAM DESIGN OF ROLLER-FOLLOWER CAM MECHANISM CONSIDERING KINEMATIC AND DYNAMIC REQUIREMENTS

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#### SUMMARY

In this paper, a new method to design cam mechanisms with translating roller follower using Matlab and Inventor software is presented. The minimization of the cam size and the contact stress can be determined by controlling design parameters, such as the cam base circle radius, the follower face width and the follower offset. During the design procedure, a number of constraints regarding the pressure angle and the contact stress are taken into account. The finite element aproach is used to perform the analysis. A specific design case has been done to prove the practical applicability of this new approach.

Keywords: Cam profile, Cam design, Synthesis, Cam follower-mechanism, FEA

#### INTRODUCTION

A cam may be defined as a machine element having a curved outline or a curved groove, which, by its oscillation or rotation motion, gives a predetermined specified motion to another element called the follower. Cam mechanism with roller follower is used widely in many classes of machines because due to the cam and follower it is possible to obtain an unlimited variety of motions. However, when a motion of follower is given arbitrarily, designers are sometime very difficult to build the cam profile to satisfy kinematic and dynamic characteristics, such as displacement, velocity, acceleration of follower, pressure angle, and even contact stress. There are a number of documents presenting about how to construct these profiles by using normal functions, such as Cycloid function, Sin function...etc[1.2]. There are many new approaches executed to improve smooth of the cam profile, but these methods are still limited to kinematic requirements [3]. The paper presents a new method to design cam mechanisms with translating roller follower concerning both kinematic and dynamic requirements, using Matlab and Inventor software. Firstly, the

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basic principle of designing a cam profile is still used to determine the coordinates of points at the center of the follower, then coordinates of points at the cam profile. NURBS is used to construct a smooth curve from these separate points. Once the function of cam profile is absolutely constructed by using NURBS curve, a MATLAB program will be done to demonstrate the curve and to control the kinematic conditions. The cam profile then can be exported into a CAD software, such as Inventor. Finally, a modeling of cam is expressed to simulate the cam mechanism on the Inventor software. Also a process of finite element analysis will be executed to control dynamic requirements. Furthermore, the results of the design process using this method can be used to manufacture cam on CNC machines. An example cam design with translating roller follower from a given displacement of follower has been done to prove the practical applicability of this new approach.

## CAM PROFILE DESIGN PRINCIPLE FROM A GIVEN DISPLACEMENT OF THE FOLLOWER

The family of pitch profile can be determined from the parametric equation [1].

$$F(x,y,\phi) = (x - x_p)^2 + (y - y_p)^2 - r_f^2 = 0$$
(1)

Where  $(x_p, y_p)$  are coordinates of points at the center of the follower, as functions of the variable  $\varphi$ ; r<sub>f</sub> is radius of roller.

$$x_{p} = (k+s)\sin\varphi + e.\cos\varphi$$
  

$$y_{p} = (k+s)\sin\varphi - e.\sin\varphi$$
;  

$$k = \sqrt{(r_{b} + r_{f})^{2} - e^{2}}$$
(2)

k is the distance parallel to the direction of motion between center of the Cam and center of the follower.

The coordinates of points at the cam profile can be calculated with the following equations [2].

$$x = x_{p} \pm r_{f} \left(\frac{dy_{p}}{d\varphi}\right) \left[ \left(\frac{dx_{p}}{d\varphi}\right)^{2} + \left(\frac{dy_{p}}{d\varphi}\right)^{2} \right]^{-1/2};$$
  

$$y = y_{p} - \left(x - x_{p}\right) \frac{\left(\frac{dx_{p}}{d\varphi}\right)}{\left(\frac{dy_{p}}{d\varphi}\right)}$$
(3)

During the synthesis procedure the following functional constraints are imposed:

1) The maximum value of the pressure angle must be smaller than the maximum permitted:  $\alpha \leq [\alpha]$ . The lower the value of pressure angle, the best the transmitted force will be transformed into the motion of the follower. If the pressure angle is too high, the follower sliding or friction will be increased.

The pressure angle can be calculated by [1.3]:

$$tg\alpha = \frac{v-e}{s+\sqrt{(r_b+r_f)^2-e^2}}$$
(4)

Where: s is displacement of the follower, v is velocity of the follower,  $r_b$  is the base circle; e is eccentricity.

2) The roller follower radius  $r_f$  is always smaller than smallest absolute value of local radius of curvature  $|1/\rho| < 1/r_f$ , that keep the contact between the cam and roller follower. If we disregard for existence of pressure angle, the minimum cam size can be determined from radius of curvature for roller follower [1.3].

$$\rho_{p} = \frac{\left\{ \left[ (r_{b} + r_{f})^{2} + s \right]^{2} \right\}^{3/2}}{\left[ (r_{b} + r_{f})^{2} + s \right]^{2} + 2v^{2} - a \left[ (r_{b} + r_{f}) + s \right]}$$
(5)

Where: a is acceleration of the follower

3) The offset e must satisfy the constraints: 0 < e < s.

4) The contact stress of any point at the cam curve is always smaller than permitted contact stress for the cam:  $\sigma_{max} \leq [\sigma]$ .

CCAM PROFILE SMOOTHING BY USING NURBS CURVESAM PROFILE SMOOTHING BY USING NURBS CURVES

In the section, NURBS curve is used to construct complex and smooth Cam profile with many constraints about kinematics, dynamics, such as displacement, velocity, acceleration, jerk of follower and pressure angle of the cam. NURBS is a flexible function used widely on applications of CAD to construct complex curves [4.5]. While most basic curves, such as the circle, are not represented by B-spline curves and other parametric polynomial curves [5]. The NURBS can be defined by generalizing Bspline curve of degree p:

$$C^{w}(u) = \sum_{i=0}^{n} N_{i,p}(u) P_{i}^{w}$$
  
=  $\sum_{i=0}^{h} N_{i,p}(u) \begin{bmatrix} w_{i} x_{i} \\ w_{i} y_{i} \\ w_{i} z_{i} \\ w_{i} \end{bmatrix}$ ; (0 ≤ u ≤ 1) (6)

Where:  $P_i^w = (w_i x_i, w_i y_i, w_i z_i)$  are control points; i = 0, 1, 2, ..., h, (h+1) are number of control points,  $N_{i,p}(u)$  are B-spline basis functions of degree p,  $w_i$  is weights, u are knots.

$$U = \left\{\underbrace{0,...,0}_{p+1}, u_{p+1}, ..., u_{e-p-1}, \underbrace{1,...,1}_{p+1}\right\}$$
(7)

is knot vector

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$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u); N_{i,0}(u)$$

$$= \begin{cases} 1 & u_i \le u \le u_{i+1} \\ 0 & u \notin [u_i & u_{i+1}] \end{cases}$$
(8)

$$R_{i,p}(u) = \frac{\sum_{i=0}^{h} N_{i,p}(u) w_i P_i}{\sum_{i=0}^{h} N_{i,p}(u) w_i} \implies C(u) = \sum_{i=0}^{h} R_{i,p}(u) P_i$$

 $R_{i,p}(u)$  are called B-spline basic Rational functions.

Suppose there is a data set of (n+1) data {D<sub>0</sub>( $x_0$ ,  $y_0$ ),..., D<sub>n</sub>( $x_n$ , $y_n$ )}, An NURBS curve of degree p is constructed from the data as using approximate method. It always get through the beginning point and the ending point, therefore the sum of the squared errors between the point of initial data and its corresponding point at the NURBS curve can be detected [3]:

$$f = f(P_0, P_1, ..., P_h, w_0, w_1, ..., w_h)$$
  
=  $\sum_{i=0}^{n} \left| \frac{\sum_{j=0}^{h} N_{j,p}(t_i) w_j P_j}{\sum_{j=0}^{h} N_{j,p}(t_i) w_j} - D_i \right|^2$  (10)

Where:  $t_i$  is parameter of initial data's point i and can be calculated by the chord length (9) method:  $t_0 = 0$ ;  $t_n = 1$ .

### FINITE ELEMENT ANALYSIS PROCEDURE

Finite element method is one of the most accepted and widely used tools for the solution of linear and non linear partial differential equations which arises during the mathematical modeling of various processes [6].

Once the function of cam profile is absolutely constructed by using NURBS curve, a MATLAB program has been done to demonstrate the curve and to test the kinematic conditions. The cam profile then can be exported into a CAD software, such as Inventor. Finally, a modeling of cam is expressed to simulate the cam mechanism. Also a process of finite element analysis will be executed. The cam design flowchart was applied in the paper as shown in Fig.1.





Permitted max. pressure angle is 25deg Cam base radius range from 60 to 120mm Permitted max. contact stress is 1500N/mm<sup>2</sup> Fig.2. Kinematic and dynamic requirements

### **RESULTS AND DISCUSSION**

In this section, an example cam design is exhibited with translating roller follower from a given follower displacement  $S(\varphi)$  as shown in Fig.2.

The input parameters are set:  $r_b = 80$ mm; e = 20mm and  $r_f = 15$ mm. The NURBS curve of degree p=3, i=90points and h=80. The cam profile and pressure angle are shown as in Fig.3. The results indicate that the maximum error  $f_{80}^{\text{max}} = 0.0587$ . The pressure angles range from 4.75deg [min] to 12.15deg [max], the maximum radius of curvature is 24.03 mm. All of these results absolutely satisfy the kinematic requirements.

Considering kinematic requirements the displacement, velocity and acceleration of the follower are determined as shown in Figure 4.

To perform finite element analysis of roller

follower, the solid model of the same is essential. Figure 5 shows a solid model of roller follower. The accuracy of the FEM depends on the density of the mesh used in the analysis; the outer surface of cam loaded by the follower so that to obtain the correct in the region of high stress.

The results of finite element analysis, as shown in Fig.6, indicate that the maximum contact stress, according to Von-mises Stress, at surface of cam is 6.4MPa with a pressure load 100N in rise cycle at angle of 19.5°, respectively. It absolutely satisfies the given dynamic requirements.



Fig.3. Cam profile and pressure angle of the cam



Fig.4. The follower motion diagrams



Fig.5. 3D model of the cam

### CONCLUSIONS

For design cam mechanisms with translating roller follower using Matlab and Inventor software have been studied in this paper taking in the consideration the kinematic and dynamic requirements for solving the NURBS curve that proposed to satisfy the curvature and minimum cam size.



Fig.6. Contact stress at cam surface

In this approach, the contact stress distribution along the contact between the follower and the cam edge which give good agreement with the numerical analysis using F.E.M have been investigated. The results show some advantage in solving non-linear programming for designing cam mechanisms with translating roller follower.

The entire data of the design process can be used for manufacturing process on CNC machines.

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### TÓM TẮT THIẾT KẾ CƠ CẦU CAM CẦN ĐẦY ĐÁY CON LĂN VỚI SỰ TRỢ GIÚP CỦA MÁY TÍNH KHI KẾ ĐẾN CÁC YÊU CẦU VỀ ĐỘNG HỌC VÀ ĐỘNG LỰC HỌC

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Bài báo này trình bày một phương pháp mới, sử dụng phần Matlab và Inventor để thiết kế cơ cấu Cam cần đẩy đáy con lăn. Trong đó, các giá trị kích thước cam và ứng suất tiếp xúc tối thiểu được xác định bằng việc điều khiển các thông số thiết kế như bán kính vòng cơ sở, bề rộng của cam và độ lệch tâm. Trong toàn bộ quá trình thiết kế, các ràng bộc về góc áp lực và ứng suất tiếp súc sẽ được đưa vào quá trình tính toán và được xem như các điều kiện biên. Phương pháp phần tử hữu hạn được sử dụng để mô phỏng sự phân bố ứng suất tiếp xúc trên bề mặt cam. Một ví dụ thiết kế cụ thể đã được tiến hành để chứng minh được khả năng áp dụng vào thực tế của của cách tiếp cận mới này.

Từ khóa: Biên dạng cam, Thiết kế cam, Tổng hợp, Cam cần đẩy, Phân tích phần tử hữu hạn

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