DESIGN OF INDIRECT MRAS-BASED ADAPTIVE CONTROL SYSTEMS

Tran Thien Dung, Dang Van Huyen, Dam Bao Loc, Nguyen Duy Cuong*
College of Technology – TNU

SUMMARY

Direct MRAS offers a potential solution to reduce the tracking errors in the presence of uncertainties and variation in plant behavior. However, this control algorithm may fail to be robust to measurement noise. In order to solve this trouble, the indirect MRAS is introduced that permanently adjust the parameters of observers. The adaptive adjusting law is derived by applying Lyapunov theory. The adaptive algorithm that is shown in this paper is quite simple, robust and converges quickly. Performances of the controlled systems are studied through simulation in Matlab/Simulink environment. The effectiveness of the methods is demonstrated by numerical simulations.

Keywords: *Direct MRAS; Indirect MRAS;Lyapunov theory*

INTRODUCTION

The PID controller is an effective solution for most industrial control applications [1], [2]. The major problem with the fixed-gain PID controller is that the tracking error depends on plant parameter variations [4], [8], [9]. Because the selection of PID gains depends on the physical characteristics of the system to be controlled, there is no set of constant values that can be suited to every implementation when the dynamic characteristics are changing. Another problem with this controller is that the PID controlled system is sensitive to measurement noise. When the error is corrupted by noise, the noise content will be amplified by PID gains. These problems can be solved, for example, by using direct or indirect adaptive control systems that are designed based on MRAS.

The basic philosophy behind Model Reference Adaptive Systems is to create a closed loop controller with parameters that can be adjusted based on the error between the output of the system and the desired response from the reference model [1] - [3]. The control parameters converge to ideal values that cause the plant response to track the response of the reference model asymptotically with time for any bounded reference input signal.

Direct MRAS in which certain information about the plant is used directly for finding appropriate ways for convergent adaptation of the controller parameters. Direct MRAS offers a potential solution to reduce the tracking errors in the presence of uncertainties and variation in plant behavior. However, this control algorithm may fail to be robust to measurement noise [6].

Indirect MRAS in which the controller is designed based on the model of the plant. All of the parameters of the model are available for adaptation. The states and the parameters of the adjustable model converge asymptotically to those of the plant. Estimation of parameters in the model leads indirectly to adaptation of parameters in the controller. In other worlds, for indirect MRAS the adaptation mechanism modifies the system performance by adjusting the parameters of the adjustable model, by adapting the parameters of the controller. Indirect MRAS offers an effective solution to improve the control performance in the presence of parametric uncertainty and measurement noise [6], [7].

In recent decades many kinds of auto-tuning PIDs have been proposed [4], [8]. However, most PID auto-tuning methods did not pay sufficient attention to the stability of the resulting PID control systems. For instance

* *Tel: 0987 920721, Email: nguyenduycuong@tnut.edu.vn*

the tuned PID parameters did not guarantee the stability of the control system for any change [3].

In this study, design of indirect MRAS-based adaptive control systems is developed for motion system which acts on the error to reject system disturbances and measurement noise, and to cope with system parameter changes. The adaptive laws are derived based on the Lyapunov's stability theory. The structures of the indirect adaptive control systems are shown with parameter calculations in more detail. The simulation results are presented and discussed.

The process of designing is built up by flowing steps:

1. Describing the process and adjustable model.
2. Determining the differential equation for error (e).
3. Choosing a Lyapunov's function V(e).
4. Defining the conditions under which $\dot{V}(e)$ is definite negative.
5. Determining (a_m, b_m) variables.
6. Solving p_{21}, p_{22} parameters.
7. Designing the PD adaptive controller.

After all needed parameters are found out. The control system will be tested and simulated in Matlab/Simulink. Then the real setup is going to be implemented.

This paper is organized as follows: Indirect MRAS is abstractly introduced in Section II. The indirect adaptive controller designing steps based on MRAS are shown in Section III. Section IV shows the simulation results implemented in Matlab/Simulink. Conclusions are given in Section V.

MODEL REFERENCE ADAPTIVE SYSTEM

Model Reference Adaptive Systems (MRAS) are one of the main approaches to adaptive control [1] - [3]. The desired performance is expressed in terms of a reference model (a model that describes the desired input-output properties of the closed loop system). When the behavior of the controlled process differs from the "ideal behavior", which is determined by the reference model, the process is modified, either by adjusting the parameters of a controller, or by generating an additional input signal for the process based on the error between the reference model output and the system output. The aim is to let parameters converge to ideal values that result in a plant response that tracks the response of the reference model.

MIT rule, the relationship between the change in theta and the cost funtion, is one of the basic techniques of adaptive control. It can be embedded into a general scheme of circuit with MRAS structure. However, the drawback ofMIT rule based MRAS design is that there is no guarantee that there sulting closed loop system will be stable [3], [8]. To overcome this difficulty, the Lyapunov theory based MRAScan be designed, which ensures that the resulting closed loop system is stable.

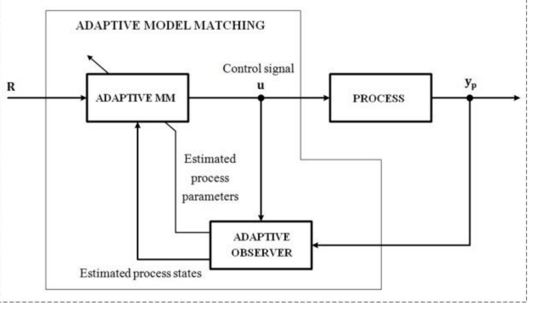

Fig 1: *The block diagram of indirect MRAS*

Fig1 shows the block diagram of the indirect MRAS, which combines an adaptive observer and adaptive MM, which stands for an adaptive model matching. The control scheme consists of two phases at each time step. The first phase consists of identifying the process dynamics by adjusting the parameters of the model. In the second phase, the adaptive MM design is implemented, not from a fixed mathematical model of the process, but from the identified model [6], [8].

DESIGN OF INDIRECT ADAPTIVE CONTROL SYSTEM

We try to design adaptive controllers for a simple system and we will encounter the problems which require more theoretical background. Simple and generally applicable adaptive laws can be found when we use the suitable Lyapunov's function.

Adjustable model

The reference model, in this case referred to as the "adjustable model", will follow the response of the process.In the following discussions the terms 'adjustable model' and "adaptive observer" are used interchangeably. The goalin process identification is to obtain a satisfactory model of areal process byobserving the process input-output behavior.

Identification of a dynamic process contains four basic steps [1], [2]. The first step is structural identification, whichallows us to characterize the structure of the mathematical model of the process to be identified. This can be done from the phenomenological analysis of the process. Next, we determine the inputs and outputs. Third step is parameteridentification. This step allows us to determine the parameters of the mathematical model of the process. Finally, the identified model is validated. When theparameters of the identified model and the process are supposed to be 'identical', the model states can be considered as estimates of the process states. When the states of the process are corrupted with noise, the structure of the adaptive observer can be used to getfiltered estimates of the process states. When the input signal itself is not very noisy, the model states will also be almost free of noise. It is important to notice that in this case the filtering is realizedwith minimum phase lag [3]. However, thisadjustable is also able to deal with unknown or time varying parameters [6].

In order to design indirect adaptive MRAS, two processes will be followed: Firstly, determining the adaptive law for variable parameters of a_m, b_m of the adjustable model.

Next, designing of PD adaptive controller based on a_m, b_m.

<u>Step 1:</u> *Determining the process and adjustable model constructions.*

Any system can be described by either its transfer function or its state space. In this case, the second order process is given by the differential equation.

$$\dot{x}_{1p} = x_{2p}$$

$$\dot{x}_{2p} = -a_p x_{2p} + b_p u_c \quad (1)$$

It would be rewritten in the state space form:

$$x_p = \begin{bmatrix} x_{1p} \\ x_{2p} \end{bmatrix}; \dot{x}_p = \begin{bmatrix} \dot{x}_{1p} \\ \dot{x}_{2p} \end{bmatrix}; \Rightarrow \dot{x}_p = A_p x_p + B_p u_c \quad (2)$$

where:

$$A_p = \begin{bmatrix} 0 & 1 \\ 0 & -a_p \end{bmatrix}; B_p = \begin{bmatrix} 0 \\ b_p \end{bmatrix}$$

Bases on the scheme of the plant, the adjustable model can be realized as following:

$$\dot{x}_{1m} = x_{2m}$$

$$\dot{x}_{2m} = -a_m x_{2m} + b_m u_c \quad (3)$$

The state space is expressed as:

$$x_m = \begin{bmatrix} x_{1m} \\ x_{2m} \end{bmatrix}; \dot{x}_m = \begin{bmatrix} \dot{x}_{1m} \\ \dot{x}_{2m} \end{bmatrix};$$

$$\Rightarrow \dot{x}_m = A_m x_m + B_m u_c \quad (4)$$

where:

$$A_m = \begin{bmatrix} 0 & 1 \\ 0 & -a_m \end{bmatrix}; B_m = \begin{bmatrix} 0 \\ b_m \end{bmatrix}$$

<u>Step 2:</u>*Deriving the error equation.*

$$e = x_p - x_m \quad (5)$$

$$\frac{de}{dt} = \frac{dx_p}{dt} - \frac{dx_m}{dt}$$

After some calculations yields:

$$\frac{de}{dt} = A_p e + A x_m + B u_c \quad (6)$$

where:

$$A = A_p - A_m, \; B = B_p - B_m$$

$$\frac{de}{dt} = A_p e + \begin{bmatrix} 0 & 0 \\ 0 & a_m - a_p \end{bmatrix} \begin{bmatrix} x_{1m} \\ x_{2m} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ b_p - b_m \end{bmatrix} u_c \quad (7)$$

$$\Leftrightarrow \frac{de}{dt} = A_p e + \begin{bmatrix} 0 & 0 \\ x_{2m} & -u_c \end{bmatrix} \begin{bmatrix} a_m - a_p \\ b_m - b_p \end{bmatrix} \quad (8)$$

Therefore:

$$\frac{de}{dt} = \mathrm{A_p} e + \psi\left(\theta - \theta^o\right) \quad (9)$$

$$\psi = \begin{bmatrix} 0 & 0 \\ x_{2\mathrm{m}} & -u_{\mathrm{c}} \end{bmatrix}, \theta = \begin{bmatrix} a_{\mathrm{m}} \\ b_{\mathrm{m}} \end{bmatrix}, \theta^o = \begin{bmatrix} a_{\mathrm{p}} \\ b_{\mathrm{p}} \end{bmatrix}$$

$$\dot{e}_1 = e_2 \Rightarrow e^{\mathrm{T}} = \begin{bmatrix} e_1 & e_2 \end{bmatrix} \quad (10)$$

$$e_1 = x_{1\mathrm{p}} - x_{1\mathrm{m}}, e_2 = x_{2\mathrm{p}} - x_{2\mathrm{m}} \quad (11)$$

Step 3: *Choosing Lyapunov's function* $V(e)$.

$$V(e) = \frac{1}{2}\gamma e^T P e + (\theta - \theta^o)^T E(\theta - \theta^o) \quad (12)$$

where:

P is an arbitrary definite positive symmetrical matrix; E is a diagonal matrix with positive elements which determine the speed of adaptation.

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}; E = \begin{bmatrix} e_{11} & 0 \\ 0 & e_{22} \end{bmatrix}$$

Step 4: *Determine the conditions under which* $\dot{V}(e)$ *is definite negative.*

From the chosen Lyapunov equation (12):

$$V(e) = \frac{1}{2}\left[\gamma e^T P e + (\theta - \theta^o)^T E(\theta - \theta^o)\right]$$

$$\frac{dV(e)}{dt} = \frac{1}{2}\gamma\left(\frac{de^T}{dt} P e + e^T P \frac{de}{dt}\right) \quad (13)$$

$$+\frac{1}{2}\left(\frac{d(\theta - \theta^o)^T}{dt} E(\theta - \theta^o) + (\theta - \theta^o)^T E \frac{d(\theta - \theta^o)}{dt}\right)$$

$$= \frac{1}{2}\gamma\left(e^T A_p P e + e^T P A_p{}^T e\right)$$

$$+\left[\gamma\left(\theta - \theta^o\right)^T \psi^T P e + \left(\theta - \theta^o\right)^T E \frac{d\theta}{dt}\right] \quad (14)$$

$$\Leftrightarrow \frac{dV(e)}{dt} = \frac{1}{2}\gamma e^T\left(P A_p{}^T + A_p P\right) e$$

$$+\left(\theta - \theta^o\right)^T\left[\gamma \psi^T P e + E \frac{d\theta}{dt}\right] \quad (15)$$

Let: $\left(P A_p{}^T + A_p P\right) = -Q$ where Q is positive definite.

$$\frac{dV(e)}{dt} = -\frac{1}{2}\gamma e^T Q e + \left(\theta - \theta^o\right)^T\left[\gamma \psi^T P e + E \frac{d\theta}{dt}\right]$$

Assume that the matrix A_p belongs to a stable system, it will follow the theorem of Malkin that Q and P are positive definite matrices.

It implies that:

$$\frac{dV(e)}{dt} = -\frac{1}{2}\gamma e^T Q e \quad (16)$$

The result of the adjustment law to be:

$$\gamma \psi^T P e + E \frac{d\theta}{dt} = 0; \frac{d\theta}{dt} = \begin{bmatrix} \frac{d\theta_1}{dt} \\ \frac{d\theta_2}{dt} \end{bmatrix} = \begin{bmatrix} \frac{da_m}{dt} \\ \frac{db_m}{dt} \end{bmatrix} \quad (17)$$

Step 5: *Solving equation* (17) *to figure out* a_m, b_m

$$\gamma \psi^T P e + E \frac{d\theta}{dt} = 0$$

$$\Rightarrow E \frac{d\theta}{dt} = -\gamma \psi^T P e \Leftrightarrow \frac{d\theta}{dt} = -\gamma E^{-1} \psi^T P e \quad (18)$$

$$= -\gamma \begin{bmatrix} e'_{22} & 0 \\ 0 & e'_{11} \end{bmatrix} \begin{bmatrix} 0 & x_{2m} \\ 0 & -u_c \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

$$\Leftrightarrow \frac{d\theta}{dt} = -\gamma \begin{bmatrix} e'_{22} & 0 \\ 0 & e'_{11} \end{bmatrix} \begin{bmatrix} x_{2m} p_{21} & x_{2m} p_{22} \\ -u_c p_{21} & -u_c p_{22} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

$$\frac{d\theta_1}{dt} = \frac{da_m}{dt} = -\gamma e'_{22}\left(p_{21} e_1 + p_{22} e_2\right) x_{2m} \quad (19)$$

$$\frac{d\theta_2}{dt} = \frac{db_m}{dt} = \gamma e'_{11}\left(p_{21} e_1 + p_{22} e_2\right) u_c \quad (20)$$

Finally, the variations of the adjustable model can be recognized as below:

$$a_m = -\gamma_1 \int \left(p_{21} e_1 + p_{22} e_2\right) x_{2m} dt + a_m(0) \quad (21)$$

$$b_m = \gamma_2 \int \left(p_{21} e_1 + p_{22} e_2\right) u_c dt + b_m(0) \quad (22)$$

Step 6: *Determining* p_{21}, p_{22} *parameters*

p_{21}, p_{22} are elements of the matrix P, obtained from the solution of the Lyapunov's equation.

If Q is positive definite, so let Q to be a chosen element. Considers that the process is varied slowly, this implies that:

$A_p \approx A_m$.

$$A_p \approx A_m = \begin{bmatrix} 0 & 1 \\ 0 & -a_m \end{bmatrix}; Q = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}$$

From equation: $\left(PA_p{}^T + A_p P\right) = -Q$

$$\Rightarrow \begin{bmatrix} p_{12} & -a_m p_{12} \\ p_{22} & -a_m p_{22} \end{bmatrix} + \begin{bmatrix} p_{21} & p_{22} \\ -a_m p_{21} & -a_m p_{22} \end{bmatrix} = -Q \quad (23)$$

$$\Rightarrow \begin{bmatrix} p_{12} + p_{21} & -a_m p_{12} + p_{22} \\ p_{22} - a_m p_{21} & -2a_m p_{22} \end{bmatrix} = -\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}$$

$$\Rightarrow p_{21} = \frac{1}{a_m}\left(\frac{1}{2a_m} + q_{21}\right); p_{22} = \frac{1}{2a_m} \quad (24)$$

Step 7:*Designing PD adaptive controller.*

A second-order process, which has already existed an I-part itself, is controlled with the aid of a PD-controller. The parameters of this controller are K_p and K_d Variations in the process parameters b_p and a_p can be compensated for by variations in K_p and K_d. We are going to find the form of the adjustment laws for K_p and K_d.

In order to design PD adaptive controller, the model matching method (MMM)is applied. It is quite simple, but the quality can be acceptable.

The total feedback system of the second order process expressed in term of transfer function is:

$$G(s) = \frac{b_p.K_p}{s^2 + \left(a_p + b_p.K_d\right)s + b_p.K_p} \quad (25)$$

The desired performance of the complete feedback system is described by the transfer function:

$$T(s) = \frac{\omega_o{}^2}{s^2 + 2\xi\omega_o s + \omega_o{}^2} \quad (26)$$

where:

ω_o is a specific frequency.

ξ is a damping factor.

For a continuous-time linear adjustable model described by

$$G(s) = \frac{b_p.K_p}{s^2 + \left(a_p + b_p.K_d\right)s + b_p.K_p} \quad (27)$$

with a cost functional defined as

$$J = \int_0^\infty |\varepsilon(t)| dt$$

where

$$\varepsilon = R - x_m \quad (28)$$

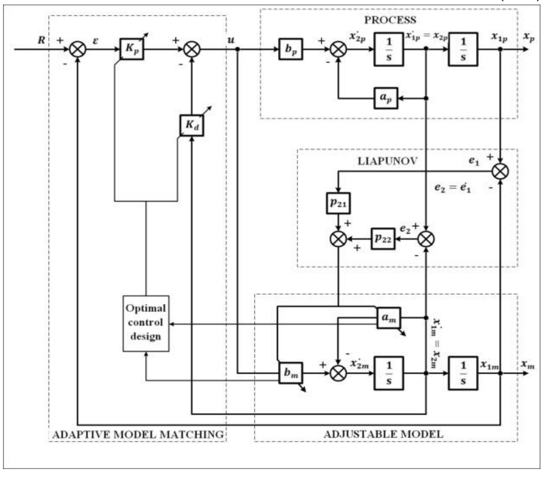

Fig 2: *Indirect adaptive controlled system*

ξ causes J, a cost function, is minimum. Bases on that, the optimal damping factor can be found out. $J = 0.7$ is corresponding to an overshoot of 5% and is optimal for industrial plants, in the field of the measurement and control.

From Eq(26) and Eq(27) a system of equations is established as following:

$$\begin{cases} b_p.K_p = \omega_o{}^2 \\ a_p + b_p.K_d = 2\xi\omega_o \end{cases} \quad (29)$$

After some calculations:

$$K_p = \frac{\omega_o{}^2}{b_p}; K_d = \frac{2\xi\omega_o - a_p}{b_p} \quad (30)$$

Setting $\omega_o = 50; \xi = 0.7$

However, paramters a_p, b_p are unknown, with adjustable model, we have: $a_m \approx a_p, b_m \approx b_p$, the equation (33) becomes:

$$K_p = \frac{2500}{b_m}; K_d = \frac{70 - a_m}{b_m} \quad (31)$$

SIMULATION RESULTS

After all parameters have already been determined in equations: $(21),(22),(31)$

$$a_m = -\gamma_1 \int \left(p_{21}e_1 + p_{22}e_2\right) x_{2m} dt + a_m(0)$$

$$b_m = \gamma_2 \int \left(p_{21}e_1 + p_{22}e_2\right) u_c dt + b_m(0)$$

$$K_p = \frac{2500}{b_m}; K_d = \frac{70 - a_m}{b_m}$$

$$Q = \begin{bmatrix} 10 & 15 \\ 15 & 10 \end{bmatrix}; \omega_o = 50; \xi = 0.7; a_m(0) = 0; b_m = 0$$

For real setup, there always exits noises acting on the process or measurement noises. Therefore, during the simulating process with Matlab/Simulink, noises are added (shown in Fig 5)

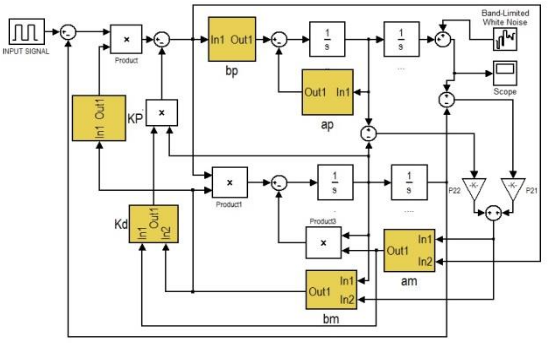

Fig 3. *The control structure implemented in Matlab/Simulink*

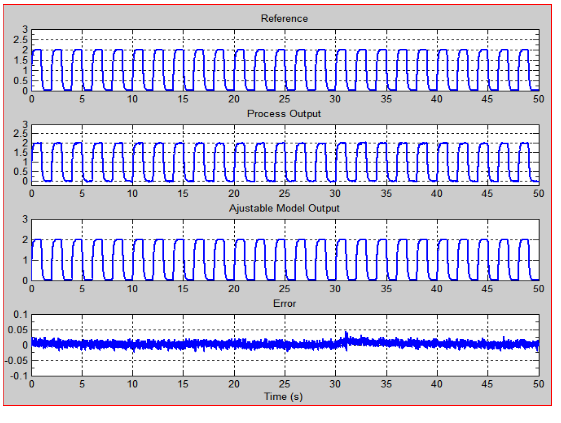

Fig 4: *Responses of process, adjustable model with the process parameter changes are added at* $t = 15(s); t = 30(s)$

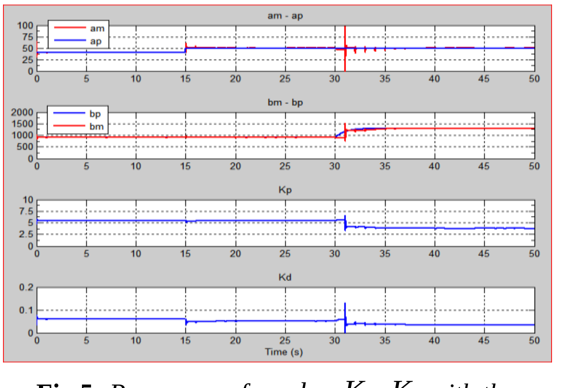

Fig 5: *Responses of* a_m, b_m, K_p, K_d *with the process parameter changes are added at* $t = 15(s); t = 30(s)$.

Discussion

- The stability conditions found with the method of Lyapunov are sufficient conditions, they are not necessary.
- The speed of adaptation, which can be varied by the adaptive gains ($e'_{22}; e'_{11}$) may in principle be chosen freely. In a practical system the adaptive gains are limited.
- The structure of the adjustable model depends on the chosen order, which is used for the identification.

CONCLUSION

This paper covers the process of designing indirect MRAS based on adaptive control systems for second order plants. The adaptive laws are derived based on the Lyapunov's stability theory. The fast adaptive schemes are proposed that continuously adjust the parameters in the controllers and/or observers. They have the advantages of the adaptive systems - quickly compensating the disturbances that can appear in the system. They are robust to changing system parameters. The way to design PD controller is simple, but it provides a good performance. The proposed adaptive control systems were tested through simulation in Matlab/Simulink environment.

REFERENCE

1. Astrom, K. J., Wittenmark, B., *Computer-Controlled Systems – Theory and Design*, Third Edition, Prentice Hall Information and System ciences Series, Prentice Hall, Upper Saddle River, 1997.

2. Landau, Y. D., *Control and Systems Theory - Adaptive Control – The Model Reference Approach*, Marcel Dekker, 1979.

3. Van Amerongen, J., *Intelligent Control (part 1)-MRAS*, Lecture notes, University of Twente, The Netherlands, March 2004.

4. R. Burkan, "Modellingof bound estimation laws and robust controllers for robustness to parametric uncertainty for control of robot manipulators", *Journal of Intelligent and Robotic Systems*, Vol. 60, pp. 365–394, 2010.

5. Pankaj, K., Kumar, J.S. and Nema, R.K. Comparative Analysis of MIT Rule and Lyapunov

Rule in Model Reference Adaptive Control Scheme, *Innovative Systems Design and Engineering*, 2(4) , pp 154- 162, 2011.
6. Nguyen DuyCuong, Advanced Controllers for Electromechanical Motion Systems, PhD thesis, University of Twente, Enschede, The Netherlands, 2008.
7. D. Simon, "Kalman filtering with state constraints- a survey of linear and nonlinear algorithm", *Control Theory and Application*, IET, Vol. 4, No. 8, pp. 1303 – 1318, 2010.
8. P. Swarnkar, S. Jain, R. K. Nema, "Application of Model Reference Adaptive Control Scheme To Second Order System Using MIT Rule", *International Conference on Electrical Power and Energy Systems (ICEPES-2010)*, MANIT, Bhopal, India, 2010.
9. Kuo-Kai Shyu, Ming-Ji Yang, Yen-Mo Chen, Yi-Fei Lin, "Model Reference Adaptive Control Design for A Shunt Active-Power-Filter System", *IEEE Transactions on Industrial Electronics*, Vol. 55, No. 1, pp. 97-106, 2008.

TÓM TẮT

THIẾT KẾ HỆ THỐNG ĐIỀU KHIỂN THÍCH NGHI DỰA TRÊN PHƯƠNG PHÁP MRAS GIÁN TIẾP

Trần Thiện Dũng, Đặng Văn Huyên,
Đàm Bảo Lộc, Nguyễn Duy Cương*
Trường Đại học Kỹ thuật Công nghiệp - ĐH Thái Nguyên

Phương pháp điều khiển thích nghi theo mô hình mẫu trực tiếp thể hiện ưu điểm khi thông số của đối tượng điều khiển không rõ và thay đổi. Tuy nhiên khi áp dụng thuật toán này hệ thống điều khiển có thể bị mất ổn định do tác động của nhiễu đo lường. Để khắc phục hạn chế của điều khiển thích nghi trực tiếp bài báo đề xuất bộ điều khiển thích nghi gián tiếp theo mô hình mẫu theo đó thông số của bộ quan sát được hiệu chỉnh liên tục trong quá trình làm việc. Luật hiệu chỉnh thích nghi nhận được bằng cách sử dụng lý thuyết ổn định Lyapunov với ưu điểm đơn giản, hội tụ nhanh và ổn định. Ưu điểm của hệ thống điều khiển đề xuất được đánh giá thông qua mô phỏng sử dụng Matlab/Simulink.
Từ khóa: *MRAS trực tiếp; MRAS gián tiếp; Lý thuyết ổn định Lyapunov*

Ngày nhận bài:20/6/2015; Ngày phản biện:06/7/2015; Ngày duyệt đăng: 30/7/2015
Phản biện khoa học: *GS. Horst Puta – Đại học Ilmenau – CHLB Đức*

* *Tel: 0987 920721, Email: nguyenduycuong@tnut.edu.vn*