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Virtual Topology and Functor Geometry

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Fred Van Oystaeyen

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Virtual Topology and Functor Geometry

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Foreword

In order to arrive at a version of Serre's global sections theorem in the noncommutative geometry of associative algebras, one is forced to introduce a noncommutative topology of Zariski type. Sheaves over such a noncommutative topology do not constitute a topos, but that is exactly the reason why sheaf theory in this generality can carry the essential noncommutative information generalizing to a satisfactory extent classical scheme theory. The noncommutativity forces, at places, a departure from set theory-based techniques resulting in a higher level of abstraction, because opens are not sets of points. Based on some intuition stemming mainly from noncommutative algebra and classical geometry, I strived for an axiomatic introduction of noncommutative topology allowing at least a minimalistic version of geometry involving actual "spaces" and not merely a mask for noncommutative algebra! Completely new problems appear already at the fundamental level, requiring new ideas that sometimes almost alienate a pure algebraist. Not all such ideas are completely developed here, often I restricted myself to bare necessities but left room for many projects ranging from the exercise level to possible research. The spirit of these notes is somewhat experimental reflecting the initial stage of the theory. This may occasionally result in a certain imbalance between novelty sections on new aspects of virtual topology and functor geometry on one hand versus well-established parts of noncommutative algebra on the other. In either case I tried to supply sufficient background material concerning localization theory or some facts on the classical lattice $L(H)$ of quantum mechanics for some Hilbert space H.

On the other hand, I included a few topics that are, at this moment, only important for some of the research projects. In recent years "research training" for so-called young researchers became a trendy topic, and several of the included projects might be viewed in such a framework; however, some projects mentioned are probably hard and essential for better development of the theory and its applications. Intrinsic problems related to sheafification over a noncommutative space are the main topic in Section 4.2 and represent the introduction of a dynamic version of noncommutative topology and geometry. Since this construction is strictly related to the "absence" of points or of "enough points" in the noncommutative spaces, the dynamic theory as defined here is an exclusively noncommutative phenomenon; it is trivialized in the commutative case where space, and its topology, is described by sets of points. While reading Section 4.3 the reader should maintain a physics point of view because a noncommutative model for "reality" is hinted at; I included some observations related to this "spaced time," resulting from recent interactions with several physicists, just as food for thought. I welcome all reactions and suggestions, for example, concerning the projects or the general philosophy of the topic.

F. Van Oystaeyen