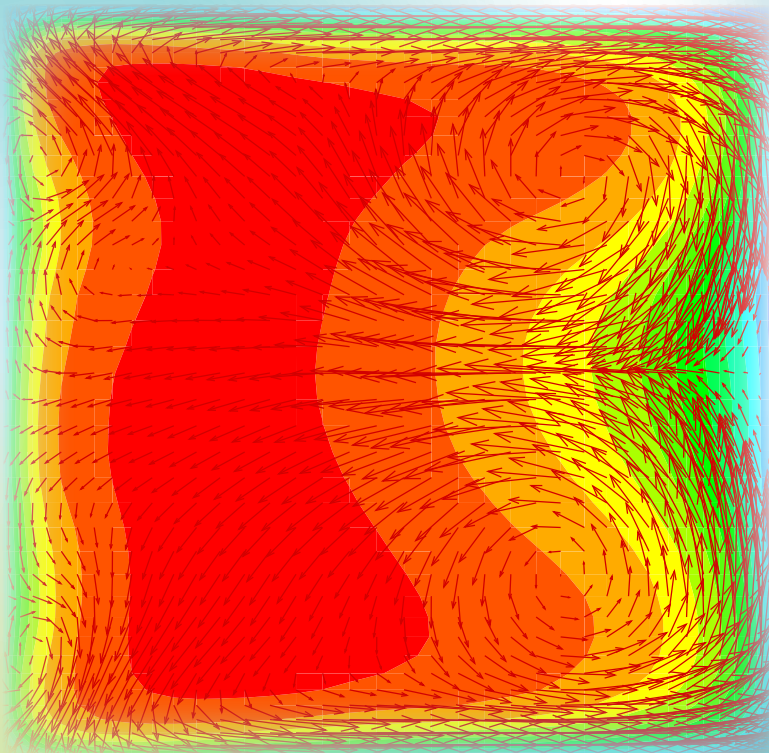


series in computational and physical processes  
in mechanics and thermal sciences

# Computational Fluid Mechanics and Heat Transfer

T H I R D   E D I T I O N



**Richard H. Pletcher**  
**John C. Tannehill**  
**Dale A. Anderson**



**CRC Press**  
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