
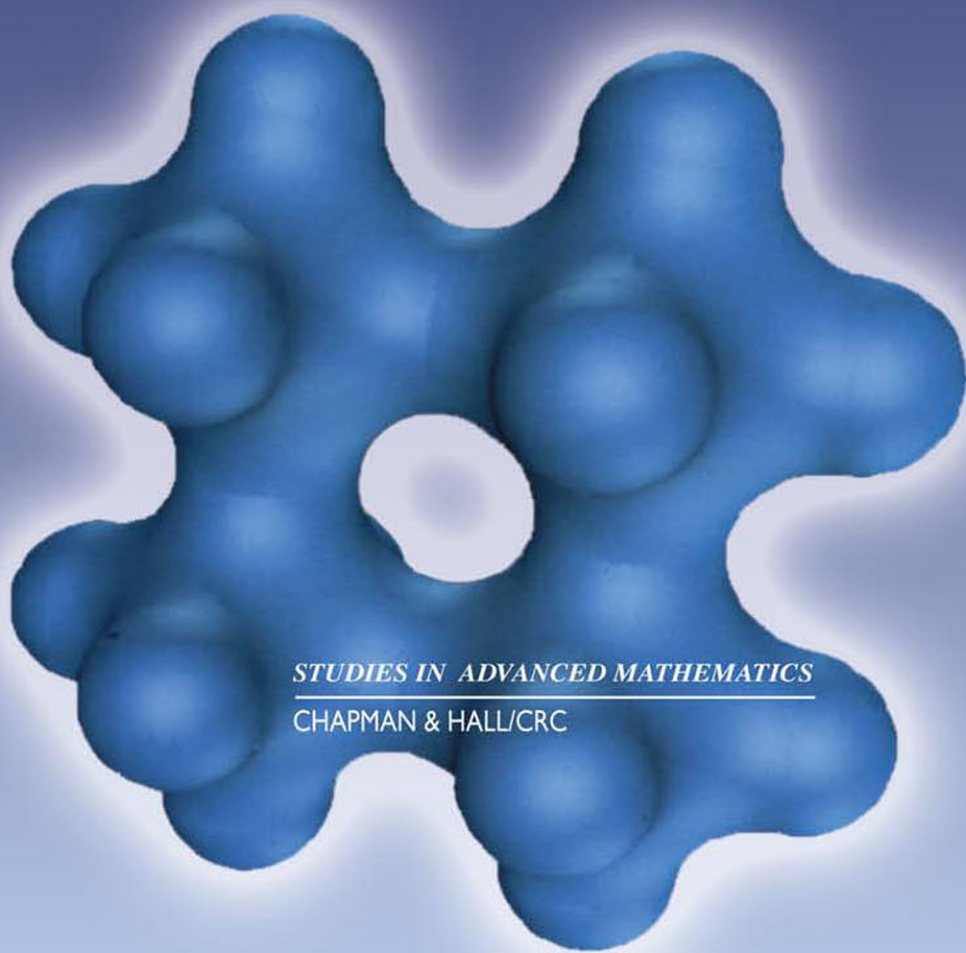


Keith Burns
Marian Gidea



Differential Geometry and Topology

With a View to
Dynamical Systems



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Dynamical Systems

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To Peter, Sonya and Imke – K.B.

To Claudia – M.G.

Preface

This book grew out of notes from a differential geometry course taught by the second author at Northwestern University. It aims to provide an introduction, at the level of a beginning graduate student, to differential topology and Riemannian geometry. The theory of differentiable dynamics has close relations to these subjects. We introduce basic concepts from dynamical systems and try to emphasize interactions of dynamics, geometry and topology.

We have attempted to introduce important concepts by intuitive discussions or suggestive examples and to follow them by significant applications, especially those related to dynamics. Where this is beyond the scope of the book, we have tried to provide references to the literature.

We have not attempted to give a comprehensive introduction to dynamical systems as this would have required a much longer book. The reader who wishes to learn more about dynamical systems should turn to one of the textbooks in that area. Three excellent recent books, with different emphases, are the texts by Brin and Stuck (2002), by Katok and Hasselblatt (1995), and by Robinson (1998).

The illustrations in this book were produced with Adobe Illustrator, DPGraph, Dynamics Solver, Maple, and Sierpinski Curve Generator. We thank Victor Donnay, Josep Masdemont, and John M. Sullivan for permission to reproduce some of the illustrations.

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