

7th edition

Elementary Linear Algebra

Ron Larson

Thu Vlen DHKTCN-TN



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Edition

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Seventh Edition

Ron Larson

The Pennsylvania State University
The Behrend College



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