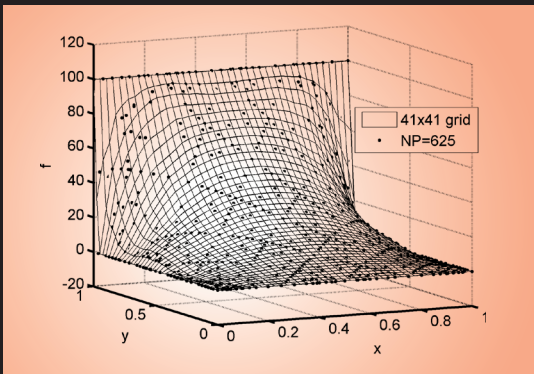
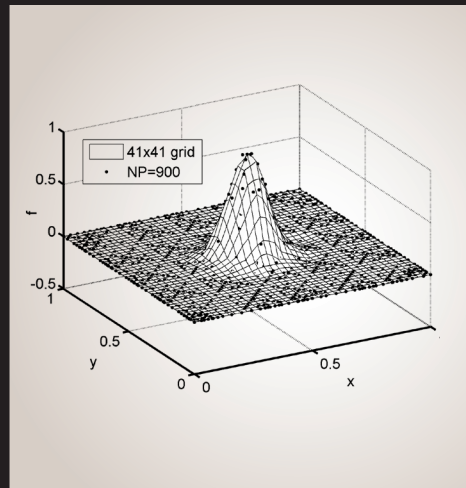
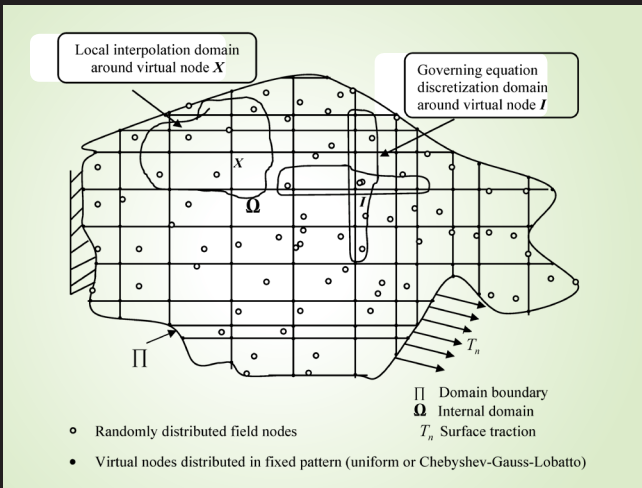


MESHLESS METHODS AND THEIR NUMERICAL PROPERTIES



HUA LI
SHANTANU S. MULAY

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Dedicated first and foremost to my motherland,
and to Duer, Anne, and my parents

Hua Li

Dedicated to my parents, and Janaki, Kaustubh,
Supriti, and little angel Aadhip

Shantanu S. Mulay

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