$$\frac{\partial x_i}{\partial p}(p,w) = -\frac{\partial^2 \Pi^*}{\partial p \partial w_i} = -\frac{\partial^2 \Pi^*}{\partial w_i \partial p} = \frac{\partial^2 E}{\partial w_i}(p,w).$$

Mathematics for Economists

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$$\{x \mid f(x) = y, x \geq 0\}$$

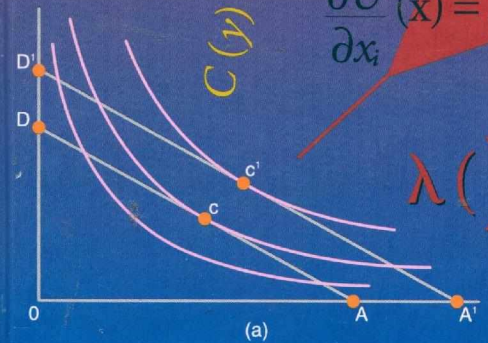
$$\dot{x} = x(r - bx)$$

$$C(y) = w \cdot x^* = \min\{w \cdot x \mid f(x) = y, x \geq 0\}$$

$$\lambda \geq 0$$

$$\frac{\partial U}{\partial x_i}(x) = \lambda p_i \quad i = 1, \ldots, n$$

$$\lambda\left(\sum_i p_i x_i - I\right) = 0$$



MATHEMATICS FOR ECONOMISTS

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Carl P. Simon
and
Lawrence Blume

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