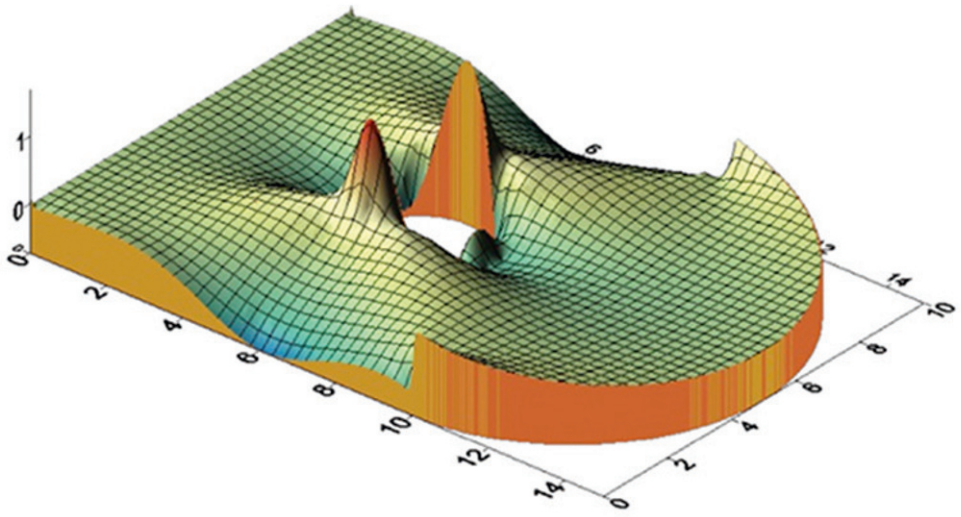
The Boundary Element Method for Plate Analysis

John T. Katsikadelis



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by
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To my wife Efi for her loving patience and support

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Foreword

At first glance this book may appear to describe yet another highly specialised method applied to the solution of plate problems, namely the one the author calls Analog Equation Method (AEM). Nothing could be further from the truth. Professor Katsikadelis has instead presented for the first time a generalised and consistent BEM for all types of plate analysis. This has been possible only because of his brilliant interpretation of the principle of virtual work.

The first two introductory chapters set the basis for the subsequent treatment.

After having set up the basic principles of boundary elements (BEM) in an elegant and consistent manner in the first chapter, the reader acquires the necessary knowledge to understand how these principles can be employed in subsequent chapters to solve many different problems.

This basic theory is then used to formulate the direct BEM for the analysis of thin plates. The benefits of having previously described the fundamentals of the method in a clear manner then become evident. Once the basic integral equations are derived, the author then demonstrates how they can be applied to write a computer programme, which results are validated through a series of comparisons.

The beauty of the approach followed by the author is that it describes how the mathematical process gives rise to equations which can be reduced to computational form for solving realistic engineering problems.

The above introductory chapters, important as they are, pale into insignificance in comparison with the rest of the book, where a series of most novel concepts are described. The author starts by describing the analysis of plates under membrane and bending forces, which leads to the equations governing buckling, large deflections and post buckling of plates. Important as these cases are, the most significant aspect is that they are solved using an original methodology based on the author's Analog Equation Method, which leads to the full analysis of a wide range of plate problems [1,2].

Few developments in Boundary Elements have been as significant as this idea of Professor Katsikadelis' and, as with all truly original ideas, it is striking in its simplicity and elegance.

To understand the AEM properly we have to refer to the basic idea behind the principle of virtual work as defined by Aristotle who stated that the behaviour of physical systems could be expressed in terms of "potentiality" and "actuality". In other words, Aristotle set up the principle of virtual "potentialities" or what we now call the principle of virtual work. While an "actual" field function is to satisfy the equations giving the problem, a "virtual" function can be more general. Usually we assume that the virtual function satisfies the same equation

as those governing the actual field, or sometimes a reduced version of those equations as in the case of the Dual Reciprocity Method [3]. Professor Katsidadelis instead gave a much wider interpretation to the virtual functions – one that would have pleased Aristotle – by stating that they do not necessarily need to satisfy the same type of governing equations of the actual problem, provided that they have the necessary degree of continuity (in the case of plate bending fourth order for instance).

The resulting Method (AEM) when combined with the use of the localised particular solutions proposed by the Dual Reciprocity Method, opens up a huge range of possibilities to Boundary Elements, some of which are presented in this book.

The part dealing with the time and non-linear analysis of plates for instance leads to a series of original formulations based on the AEM. The possibility of solving problems with membrane as well as bending forces can now be fully exploited for cases like dynamic buckling, including flutter instability and a series of applications of fundamental importance in aerospace engineering for instance. Extensions to the case of membranes, non linear materials and large deformations follow effortlessly.

Throughout the book the reader will find a clarity of exposition and consistency which allows the progression from simple to more complex problems in a stepwise fashion. This results in obtaining a full comprehension of the basic principles and how they are applied to obtain practical solutions in a way that is frequently missing in the current engineering sciences literature.

The fact that this book centres on the concept of the AEM developed by the author does not imply any restrictions as the AEM can be interpreted to be the most general version of the principle of virtual work developments ever presented in science and engineering.

Professor Katsikadelis' Method effortlessly transforms a series of complex problems into alternative problems which can be solved in BEM form using simple fundamental solutions.

An added advantage of the AEM is that it allows for the solution of a given set of problems, in the case of plates for instance, using the same type of computer programme. This generality will allow boundary elements to become more widely used for plates and shells, types of problems for which the method still lags behind the less accurate but more versatile finite element method.

Those interested in knowing more about the many contributions of Professor Katsikadelis to the solution of a wide variety of engineering problems and the development of many different ideas, ought to refer to my own appraisal of his work in reference [4]. It was while compiling that paper that I came to fully appreciate his work, including his many contributions to the analysis of plates.

The contents of the present book represent without doubt, a major development in engineering sciences.

Carlos A. Brebbia

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