Primis Online

Mathematics

The History of Mathematics: An Introduction, 6th Editi
Burton

McGraw-Hill
A Division of The McGraw-Hill Companies

McGraw–Hill Primis

ISBN: 0–390–63234–1

Text:

The History of Mathematics: An Introduction, Sixth Edition
Burton

This book was printed on recycled paper.

Mathematics

http://www.mhhe.com/primis/online/

111 MATHGEN ISBN: 0–390–63234–1

Mathematics

Contents

Burton • *The History of Mathematics: An Introduction, Sixth Edition*

Preface

Since many excellent treatises on the history of mathematics are available, there may seem little reason for writing still another. But most current works are severely technical, written by mathematicians for other mathematicians or for historians of science. Despite the admirable scholarship and often clear presentation of these works, they are not especially well adapted to the undergraduate classroom. (Perhaps the most notable exception is Howard Eves's popular account, *An Introduction to the History of Mathematics*.) There seems to be room at this time for a textbook of tolerable length and balance addressed to the undergraduate student, which at the same time is accessible to the general reader interested in the history of mathematics.

In the following pages, I have tried to give a reasonably full account of how mathematics has developed over the past 5000 years. Because mathematics is one of the oldest intellectual instruments, it has a long story, interwoven with striking personalities and outstanding achievements. This narrative is basically chronological, beginning with the origin of mathematics in the great civilizations of antiquity and progressing through the later decades of the twentieth century. The presentation necessarily becomes less complete for modern times, when the pace of discovery has been rapid and the subject matter more technical.

Considerable prominence has been assigned to the lives of the people responsible for progress in the mathematical enterprise. In emphasizing the biographical element, I can say only that there is no sphere in which individuals count for more than the intellectual life, and that most of the mathematicians cited here really did tower over their contemporaries. So that they will stand out as living figures and representatives of their day, it is necessary to pause from time to time to consider the social and cultural framework that animated their labors. I have especially tried to define why mathematical activity waxed and waned in different periods and in different countries.

Writers on the history of mathematics tend to be trapped between the desire to interject some genuine mathematics into a work and the desire to make the reading as painless and pleasant as possible. Believing that any mathematics textbook should concern itself primarily with teaching mathematical content, I have favored stressing the mathematics. Thus, assorted problems of varying degrees of difficulty have been interspersed throughout. Usually these problems typify a particular historical period, requiring the procedures of that time. They are an integral part of the text, and you will, in working them, learn some interesting mathematics as well as history. The level of maturity needed for this work is approximately the mathematical background of a college junior or senior. Readers with more extensive training in the subject must forgive certain explanations that seem unnecessary.

The title indicates that this book is in no way an encyclopedic enterprise. Neither does it pretend to present all the important mathematical ideas that arose during the vast sweep of time it covers. The inevitable limitations of space necessitate illuminating some outstanding landmarks instead of casting light of equal brilliance over the whole landscape. In keeping with this outlook, a certain amount of judgment and self-denial has to be exercised, both in choosing mathematicians and in treating their contributions. Nor was material selected exclusively on objective factors; some personal tastes and prejudices held sway. It stands to reason that not everyone will be satisfied with the choices. Some readers will raise an eyebrow at the omission of some household names of mathematics that have been either passed over in complete silence or shown no great hospitality; others will regard the scant treatment of their favorite topic as an unpardonable omission. Nevertheless, the path that I have pieced together should provide an adequate explanation of how mathematics came to occupy its position as a primary cultural force in Western civilization. The book is published in the modest hope that it may stimulate the reader to pursue the more elaborate works on the subject.

Anyone who ranges over such a well-cultivated field as the history of mathematics becomes so much in debt to the scholarship of others as to be virtually pauperized. The chapter bibliographies represent a partial listing of works, recent and not so recent, that in one way or another have helped my command of the facts. To the writers and to many others of whom no record was kept, I am enormously grateful.

New to This Edition

Readers familiar with previous editions of *The History of Mathematics* will find that this edition maintains the same overall organization and content. Nevertheless, the preparation of a sixth edition has provided the occasion for a variety of small improvements as well as several more significant ones.

The most pronounced difference is a considerably expanded discussion of Chinese and Islamic mathematics in Section 5.5. A significant change also occurs in Section 12.2 with an enhanced treatment of Henri Poincaré's career. An enlarged Section 10.3 now focuses more closely on the role of the number theorists P. G. Lejeune Dirichlet and Carl Gustav Jacobi. The presentation of the rise of American mathematics (Section 12.1) is carried further into the early decades of the twentieth century by considering the achievements of George D. Birkhoff and Norbert Wiener.

Another noteworthy difference is the increased attention paid to several individuals touched upon too lightly in previous editions. For instance, material has been added regarding the mathematical contributions of Apollonius of Perga, Regiomontanus, Robert Recorde, Simeon-Denis Poisson, Gaspard Monge and Stefan Banach.

Beyond these textual modifications, there are a number of relatively minor changes. A broadened table of contents more effectively conveys the material in each chapter, making it easier to locate a particular period, topic, or great master. Further exercises have been introduced, bibliographies brought up to date, and certain numerical information kept current. Needless to say, an attempt has been made to correct errors, typographical and historical, which crept into the earlier versions.

Acknowledgments

Many friends, colleagues, and readers—too numerous to mention individually—have been kind enough to forward corrections or to offer suggestions for the book's enrichment. I hope that they will accept a general statement of thanks for their collective contributions. Although not every recommendation was incorporated, all were gratefully received and seriously considered when deciding upon alterations.

In particular, the advice of the following reviewers was especially helpful in the creation of the sixth edition:

Rebecca Berg, Bowie State University
Henry Gould, West Virginia University
Andrzej Gutek, Tennessee Technological University
Mike Hall, Arkansas State University

Ho Kuen Ng, San Jose State University
Daniel Otero, Xavier University
Sanford Segal, University of Rochester
Chia-Chi Tung, Minnesota State University—Mankato
William Wade, University of Tennessee

A special debt of thanks is owed my wife, Martha Beck Burton, for providing assistance throughout the preparation of this edition; her thoughtful comments significantly improved the exposition. Last, I would like to express my appreciation to the staff members of McGraw-Hill for their unfailing cooperation during the course of production.

Any errors that have survived all this generous assistance must be laid at my door.

D.M.B.

CHAPTER 1

Early Number Systems and Symbols

To think the thinkable—that is the mathematician's aim.

C. J. KEYSER

1.1 Primitive Counting

A Sense of Number

The root of the term *mathematics* is in the Greek word *mathemata*, which was used quite generally in early writings to indicate any subject of instruction or study. As learning advanced, it was found convenient to restrict the scope of this term to particular fields of knowledge. The Pythagoreans are said to have used it to describe arithmetic and geometry; previously, each of these subjects had been called by its separate name, with no designation common to both. The Pythagoreans' use of the name would perhaps be a basis for the notion that mathematics began in Classical Greece during the years from 600 to 300 B.C. But its history can be followed much further back. Three or four thousand years ago, in ancient Egypt and Babylonia, there already existed a significant body of knowledge that we should describe as mathematics. If we take the broad view that mathematics involves the study of issues of a quantitative or spatial nature—number, size, order, and form—it is an activity that has been present from the earliest days of human experience. In every time and culture, there have been people with a compelling desire to comprehend and master the form of the natural world around them. To use Alexander Pope's words, "This mighty maze is not without a plan."

It is commonly accepted that mathematics originated with the practical problems of counting and recording numbers. The birth of the idea of number is so hidden behind the veil of countless ages that it is tantalizing to speculate on the remaining evidences of early humans' sense of number. Our remote ancestors of some 20,000 years ago—who were quite as clever as we are—must have felt the need to enumerate their livestock, tally objects for barter, or mark the passage of days. But the evolution of counting, with its spoken number words and written number symbols, was gradual and does not allow any determination of precise dates for its stages.

Anthropologists tell us that there has hardly been a culture, however primitive, that has not had some awareness of number, though it might have been as rudimentary as the distinction between one and two. Certain Australian aboriginal tribes, for instance, counted to two only, with any number larger than two called simply "much" or "many." South American Indians along the tributaries of the Amazon were equally destitute of number words. Although they ventured further than the aborigines in being able to count to six, they had no independent number names for groups of three, four, five, or six. In their counting vocabulary, three was called "two-one," four was "two-two," and so on. A similar system has been reported for the Bushmen of South Africa, who counted to ten $(10 = 2 + 2 + 2 + 2 + 2)$ with just two words; beyond ten, the descriptive phrases became too long. It is notable that such tribal groups would not willingly trade, say, two cows for four pigs, yet had no hesitation in exchanging one cow for two pigs and a second cow for another two pigs.

The earliest and most immediate technique for visibly expressing the idea of number is tallying. The idea in tallying is to match the collection to be counted with some easily employed set of objects—in the case of our early forebears, these were fingers, shells, or stones. Sheep, for instance, could be counted by driving them one by one through a narrow passage while dropping a pebble for each. As the flock was gathered in for the night, the pebbles were moved from one pile to another until all the sheep had been accounted for. On the occasion of a victory, a treaty, or the founding of a village, frequently a cairn, or pillar of stones, was erected with one stone for each person present.

The term *tally* comes from the French verb *tailler*, "to cut," like the English word *tailor*; the root is seen in the Latin *taliare*, meaning "to cut." It is also interesting to note that the English word *write* can be traced to the Anglo-Saxon *writan*, "to scratch," or "to notch."

Neither the spoken numbers nor finger tallying have any permanence, although finger counting shares the visual quality of written numerals. To preserve the record of any count, it was necessary to have other representations. We should recognize as human intellectual progress the idea of making a correspondence between the events or objects recorded and a series of marks on some suitably permanent material, with one mark representing each individual item. The change from counting by assembling collections of physical objects to counting by making collections of marks on one object is a long step, not only toward abstract number concept, but also toward written communication.

Counts were maintained by making scratches on stones, by cutting notches in wooden sticks or pieces of bone, or by tying knots in strings of different colors or lengths. When the numbers of tally marks became too unwieldy to visualize, primitive people arranged them in easily recognizable groups such as groups of five, for the fingers of a hand. It is likely that grouping by pairs came first, soon abandoned in favor of groups of 5, 10, or 20. The organization of counting by groups was a noteworthy improvement on counting by ones. The practice of counting by fives, say, shows a tentative sort of progress toward reaching an abstract concept of "five" as contrasted with the descriptive ideas "five fingers" or "five days." To be sure, it was a timid step in the long journey toward detaching the number sequence from the objects being counted.

Notches as Tally Marks

Bone artifacts bearing incised markings seem to indicate that the people of the Old Stone Age had devised a system of tallying by groups as early as 30,000 B.C. The most impressive example is a shinbone from a young wolf, found in Czechoslovakia in 1937; about 7 inches long, the bone is engraved with 55 deeply cut notches, more or less equal in length, arranged in groups of five. (Similar recording notations are still used, with the strokes bundled in fives, like 𝍸. Voting results in small towns are still counted in the manner devised by our remote ancestors.) For many years such notched bones were interpreted as hunting tallies and the incisions were thought to represent kills. A more recent theory, however, is that the first recordings of ancient people were concerned with reckoning time. The markings on bones discovered in French cave sites in the late 1880s are grouped in sequences of recurring numbers that agree with the numbers of days included in successive phases of the moon. One might argue that these incised bones represent lunar calendars.

Another arresting example of an incised bone was unearthed at Ishango along the shores of Lake Edward, one of the headwater sources of the Nile. The best archeological and geological evidence dates the site to 17,500 B.C., or some 12,000 years before the first settled agrarian communities appeared in the Nile valley. This fossil fragment was probably the handle of a tool used for engraving, or tattooing, or even writing in some way. It contains groups of notches arranged in three definite columns; the odd, unbalanced composition does not seem to be decorative. In one of the columns, the groups are composed of 11, 21, 19, and 9 notches. The underlying pattern may be $10 + 1$, $20 + 1$, $20 - 1$, and $10 - 1$. The notches in another column occur in eight groups, in the following order: 3, 6, 4, 8, 10, 5, 5, 7. This arrangement seems to suggest an appreciation of the concept of duplication, or multiplying by 2. The last column has four groups consisting of 11, 13, 17, and 19 individual notches. The pattern here may be fortuitous and does not necessarily indicate—as some authorities are wont to infer—a familiarity with prime numbers. Because $11 + 13 + 17 + 19 = 60$ and $11 + 21 + 19 + 9 = 60$, it might be argued that markings on the prehistoric Ishango bone are related to a lunar count, with the first and third columns indicating two lunar months.

The use of tally marks to record counts was prominent among the prehistoric peoples of the Near East. Archaeological excavations have unearthed a large number of small clay objects that had been hardened by fire to make them more durable. These handmade artifacts occur in a variety of geometric shapes, the most common being circular disks, triangles, and cones. The oldest, dating to about 8000 B.C., are incised with sets of parallel lines on a plain surface; occasionally, there will be a cluster of circular impressions as if punched into the clay by the blunt end of a bone or stylus. Because they go back to the time when people first adopted a settled agricultural life, it is believed that the objects are primitive reckoning devices; hence, they have become known as "counters" or "tokens." It is quite likely also that the shapes represent different commodities. For instance, a token of a particular type might be used to indicate the number of animals in a herd, while one of another kind could count measures of grain. Over several millennia, tokens became increasingly complex, with diverse markings and new shapes. Eventually, there came to be 16 main forms of tokens. Many were perforated with small holes, allowing them to be strung together for safekeeping. The token system of recording information went out of favor around 3000 B.C., with the rapid adoption of writing on clay tablets.

A method of tallying that has been used in many different times and places involves the notched stick. Although this device provided one of the earliest forms of keeping records, its use was by no means limited to "primitive peoples," or for that matter, to the remote past. The acceptance of tally sticks as promissory notes or bills of exchange reached its highest level of development in the British Exchequer tallies, which formed an essential part of the government records from the twelfth century onward. In this instance, the tallies were flat pieces of hazelwood about 6–9 inches long and up to an inch thick. Notches of varying sizes and types were cut in the tallies, each notch representing a fixed amount of money. The width of the cut decided its value. For example, the notch of £1000 was as large as