
Mathematics for Economics

second edition

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Preface

A major challenge in writing a book on mathematics for economists is to select the appropriate mathematical topics and present them with the necessary clarity. Another challenge is to motivate students of economics to study these topics by convincingly demonstrating their power to deal with economic problems. All this must be done without sacrificing anything in terms of the rigor and correctness of the mathematics itself.

A problem lies in the difference between the logic of the development of the mathematics and the way in which economics progresses from models of individual consumer and firm, through market models and general equilibrium, to macroeconomic models. The primary building blocks, the models of consumer and firm behavior, are based on methods of constrained optimization that, mathematically speaking, are already relatively advanced. In this book we have chosen instead to follow the logic of the mathematics. After a review of fundamentals, concerned primarily with sets, numbers, and functions, we pay careful attention to the development of the ideas of limits and continuity, moving then to the calculus of functions of one variable, linear algebra, multivariate calculus, and finally, dynamics. In the treatment of the mathematics our goal has always been to give the student an understanding of the mathematical concepts themselves, since we believe this understanding is required if he or she is to develop the ability and confidence to tackle problems in economic analysis. We have very consciously sought to avoid a "cookbook" approach.

We have tried to develop the student's problem-solving skills and motivation by working through a large number of examples and economic applications, far more than is usual in this type of book. Although the selection of these, and the order in which they are presented, was determined by the logic of the development of the mathematics rather than that of an economics course, in the end the student will have covered virtually all of the standard undergraduate mathematical economics syllabus.

Many people helped us in the preparation of this book and it is a pleasure to acknowledge our debt to them here. The following individuals read early versions of the manuscript and offered helpful suggestions, a large number of which were freely used:

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Part I

Introduction and Fundamentals

Chapter 1
Introduction

Chapter 2
Review of Fundamentals

Chapter 3
Sequences, Series, and Limits