

Walter Greiner

# Classical Mechanics

Systems of Particles and Hamiltonian Dynamics

*Second Edition*

 Springer

---

# Classical Mechanics

Second Edition

Greiner  
**Quantum Mechanics**  
An Introduction 4th Edition

Greiner  
**Quantum Mechanics**  
Special Chapters

Greiner · Müller  
**Quantum Mechanics**  
Symmetries 2nd Edition

Greiner  
**Relativistic Quantum Mechanics**  
Wave Equations 3rd Edition

Greiner · Reinhardt  
**Field Quantization**

Greiner · Reinhardt  
**Quantum Electrodynamics**  
4th Edition

Greiner · Schramm · Stein  
**Quantum Chromodynamics**  
3rd Edition

Greiner · Maruhn  
**Nuclear Models**

Greiner · Müller  
**Gauge Theory of Weak Interactions**  
4th Edition

Greiner  
**Classical Mechanics**  
Systems of Particles  
and Hamiltonian Dynamics  
2nd Edition

Greiner  
**Classical Mechanics**  
Point Particles and Relativity

Greiner  
**Classical Electrodynamics**

Greiner · Neise · Stocker  
**Thermodynamics  
and Statistical Mechanics**

---

Walter Greiner

# Classical Mechanics

Systems of Particles and  
Hamiltonian Dynamics

With a Foreword by  
D.A. Bromley

Second Edition  
With 280 Figures,  
and 167 Worked Examples and Exercises

 Springer

Prof. Dr. Walter Greiner  
Frankfurt Institute  
for Advanced Studies (FIAS)  
Johann Wolfgang Goethe-Universität  
Ruth-Moufang-Str. 1  
60438 Frankfurt am Main  
Germany  
[greiner@fias.uni-frankfurt.de](mailto:greiner@fias.uni-frankfurt.de)

---

Translated from the German *Mechanik: Teil 2*, by Walter Greiner, published by Verlag Harri Deutsch, Thun, Frankfurt am Main, Germany, © 1989

---

ISBN 978-3-642-03433-6  
DOI 10.1007/978-3-642-03434-3  
Springer Heidelberg Dordrecht London New York

e-ISBN 978-3-642-03434-3

Library of Congress Control Number: 2009940125

© Springer-Verlag Berlin Heidelberg 1992, 2010

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable to prosecution under the German Copyright Law.

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

*Cover design:* eStudio Calamar S.L., Spain

Printed on acid-free paper

Springer is part of Springer Science+Business Media ([www.springer.com](http://www.springer.com))

---

## Foreword

More than a generation of German-speaking students around the world have worked their way to an understanding and appreciation of the power and beauty of modern theoretical physics—with mathematics, the most fundamental of sciences—using Walter Greiner’s textbooks as their guide.

The idea of developing a coherent, complete presentation of an entire field of science in a series of closely related textbooks is not a new one. Many older physicians remember with real pleasure their sense of adventure and discovery as they worked their ways through the classic series by Sommerfeld, by Planck, and by Landau and Lifshitz. From the students’ viewpoint, there are a great many obvious advantages to be gained through the use of consistent notation, logical ordering of topics, and coherence of presentation; beyond this, the complete coverage of the science provides a unique opportunity for the author to convey his personal enthusiasm and love for his subject.

These volumes on classical physics, finally available in English, complement Greiner’s texts on quantum physics, most of which have been available to English-speaking audiences for some time. The complete set of books will thus provide a coherent view of physics that includes, in classical physics, thermodynamics and statistical mechanics, classical dynamics, electromagnetism, and general relativity; and in quantum physics, quantum mechanics, symmetries, relativistic quantum mechanics, quantum electro- and chromodynamics, and the gauge theory of weak interactions.

What makes Greiner’s volumes of particular value to the student and professor alike is their completeness. Greiner avoids the all too common “it follows that . . .,” which conceals several pages of mathematical manipulation and confounds the student. He does not hesitate to include experimental data to illuminate or illustrate a theoretical point, and these data, like the theoretical content, have been kept up to date and topical through frequent revision and expansion of the lecture notes upon which these volumes are based.

Moreover, Greiner greatly increases the value of his presentation by including something like one hundred completely worked examples in each volume. Nothing is of greater importance to the student than seeing, in detail, how the theoretical concepts and tools under study are applied to actual problems of interest to working physicists. And, finally, Greiner adds brief biographical sketches to each chapter covering the people responsible for the development of the theoretical ideas and/or the experimental data presented. It was Auguste Comte (1789–1857) in his *Positive Philosophy* who noted, “To understand a science it is necessary to know its history.” This is all too often forgotten in modern physics teaching, and the bridges that Greiner builds to the pioneering figures of our science upon whose work we build are welcome ones.

Greiner’s lectures, which underlie these volumes, are internationally noted for their clarity, for their completeness, and for the effort that he has devoted to making physics

an integral whole. His enthusiasm for his sciences is contagious and shines through almost every page.

These volumes represent only a part of a unique and Herculean effort to make all of theoretical physics accessible to the interested student. Beyond that, they are of enormous value to the professional physicist and to all others working with quantum phenomena. Again and again, the reader will find that, after dipping into a particular volume to review a specific topic, he or she will end up browsing, caught up by often fascinating new insights and developments with which he or she had not previously been familiar.

Having used a number of Greiner's volumes in their original German in my teaching and research at Yale, I welcome these new and revised English translations and would recommend them enthusiastically to anyone searching for a coherent overview of physics.

Yale University  
New Haven, Connecticut, USA

D. Allan Bromley  
Henry Ford II Professor of Physics

---

## Preface to the Second Edition

I am pleased to note that our text *Classical Mechanics: Systems of Particles and Hamiltonian Dynamics* has found many friends among physics students and researchers, and that a second edition has become necessary. We have taken this opportunity to make several amendments and improvements to the text. A number of misprints and minor errors have been corrected and explanatory remarks have been supplied at various places.

New examples have been added in Chap. 19 on canonical transformations, discussing the harmonic oscillator (19.3), the damped harmonic oscillator (19.4), infinitesimal time steps as canonical transformations (19.5), the general form of Liouville's theorem (19.6), the canonical invariance of the Poisson brackets (19.7), Poisson's theorem (19.8), and the invariants of the plane Kepler system (19.9).

It may come as a surprise that even for a time-honored subject such as Classical Mechanics in the formulation of Lagrange and Hamilton, new aspects may emerge. But this has indeed been the case, resulting in new chapters on the "Extended Hamilton–Lagrange formalism" (Chap. 21) and the "Extended Hamilton–Jacobi equation" (Chap. 22). These topics are discussed here for the first time in a textbook, and we hope that they will help to convince students that even Classical Mechanics can still be an active area of ongoing research.

I would especially like to thank Dr. Jürgen Struckmeier for his help in constructing the new chapters on the Extended Hamilton–Lagrange–Jacobi formalism, and Dr. Stefan Scherer for his help in the preparation of this new edition. Finally, I appreciate the agreeable collaboration with the team at Springer-Verlag, Heidelberg.

Frankfurt am Main  
September 2009

Walter Greiner





---

## Preface to the First Edition

Theoretical physics has become a many faceted science. For the young student, it is difficult enough to cope with the overwhelming amount of new material that has to be learned, let alone obtain an overview of the entire field, which ranges from mechanics through electrodynamics, quantum mechanics, field theory, nuclear and heavy-ion science, statistical mechanics, thermodynamics, and solid-state theory to elementary-particle physics; and this knowledge should be acquired in just eight to ten semesters, during which, in addition, a diploma or master's thesis has to be worked on or examinations prepared for. All this can be achieved only if the university teachers help to introduce the student to the new disciplines as early on as possible, in order to create interest and excitement that in turn set free essential new energy.

At the Johann Wolfgang Goethe University in Frankfurt am Main, we therefore confront the student with theoretical physics immediately, in the first semester. Theoretical Mechanics I and II, Electrodynamics, and Quantum Mechanics I—An Introduction are the courses during the first two years. These lectures are supplemented with many mathematical explanations and much support material. After the fourth semester of studies, graduate work begins, and Quantum Mechanics II—Symmetries, Statistical Mechanics and Thermodynamics, Relativistic Quantum Mechanics, Quantum Electrodynamics, Gauge Theory of Weak Interactions, and Quantum Chromodynamics are obligatory. Apart from these, a number of supplementary courses on special topics are offered, such as Hydrodynamics, Classical Field Theory, Special and General Relativity, Many-Body Theories, Nuclear Models, Models of Elementary Particles, and Solid-State Theory.

This volume of lectures, *Classical Mechanics: Systems of Particles and Hamiltonian Dynamics*, deals with the second and more advanced part of the important field of classical mechanics. We have tried to present the subject in a manner that is both interesting to the student and easily accessible. The main text is therefore accompanied by many exercises and examples that have been worked out in great detail. This should make the book useful also for students wishing to study the subject on their own.

Beginning the education in theoretical physics at the first university semester, and not as dictated by tradition after the first one and a half years in the third or fourth semester, has brought along quite a few changes as compared to the traditional courses in that discipline. Especially necessary is a greater amalgamation between the actual physical problems and the necessary mathematics. Therefore, we treat in the first semester vector algebra and analysis, the solution of ordinary, linear differential equations, Newton's mechanics of a mass point, and the mathematically simple mechanics of special relativity.

Many explicitly worked-out examples and exercises illustrate the new concepts and methods and deepen the interrelationship between physics and mathematics. As a